# Formal Software Specification Using RAISE

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# Introduction





## Course Aim

Introduction to formal specification

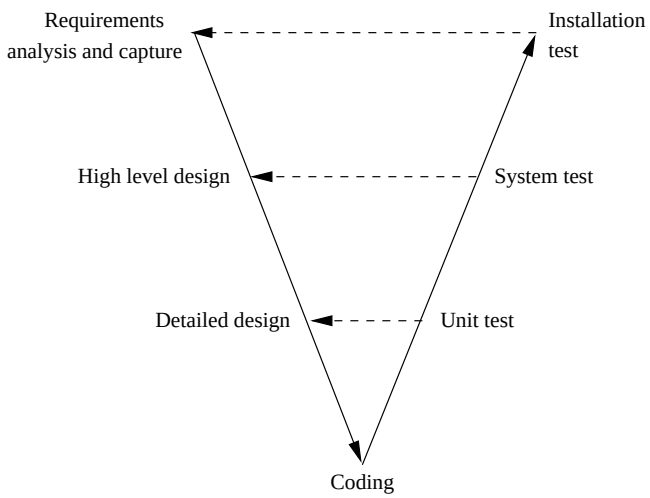
- languages
- techniques
- methods

Specific method: RAISE





## V-diagram model of software life cycle



The V-diagram illustrates the typical re-work cycles when we discover errors by testing.

We aim to *find errors earlier*.

We concentrate on the early stages:

- *requirements analysis and capture*
- *high level design*





## Specification

| specification | implementation |
|---|---|
| (abstract) | (concrete) |
| (what) | (how) |

Mathematical models describe real world

specification ——> program





## Formal

Formal specification language:

- precise syntax
- mathematical meaning (semantics)

Formality =>

- unambiguous
- formal reasoning (prove properties)





## Methods for Software Development

- Ad hoc
- Systematic
- Rigorous (R in RAISE: Rigorous)
- Formal

## What is RAISE?

**R**igorous **A**pproach to
**I**ndustrial **S**oftware **E**ngineering

**RAISE** is a product consisting of:

- a method for software development
- a formal specification language: RSL
- computer based tools

developed by:

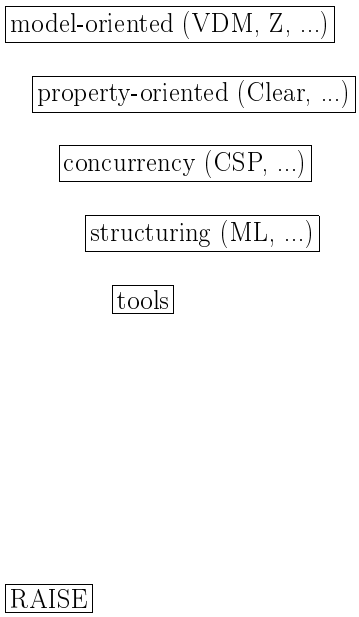
- DDC/CRI (DK)
- STL/BNR (UK)
- ICL (UK)
- NBB/ABB/SYPRO (DK)

in an ESPRIT-I project, RAISE, 1985 - 1990





## Background








## RAISE Continuation

ESPRIT-II project, **LaCoS**, 1990 - 1995

**L**arge **S**cale **C**orrect **S**ystems
Using Formal Methods

- industrial applications of RAISE
- evolution of
  RAISE method, language and tools





## LaCoS Partners

**Producers:**

CRI (DK)
SYPRO (DK)
BNR Europe (UK)

**Consumers:**

| | |
|---|---|
| BNR Europe (UK): | Network design toolset |
| Lloyd's Register (UK): | Ship engine monitoring; security |
| Bull (F): | Database; security |
| MATRA Transport (F): | Automatic train protection |
| Inisel Espacio (E): | Image processing |
| Space Software Italia (I): | Tethered satellite; air traffic control |
| Technisystems (GR): | Shipping transaction processing |

## RAISE Specification Language, RSL

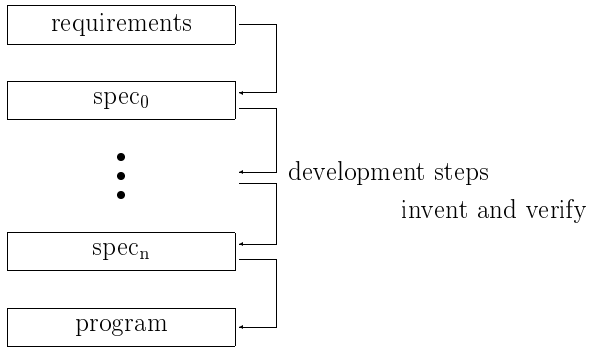
Features:

- Formal
- Wide spectrum
  - model-oriented and property-oriented
  - applicative and imperative
  - sequential and concurrent
- Structuring facilities





## RAISE Method




specification is formal (formulated in RSL)

verification may be formal





## RAISE Tools

- Storage of
  - specifications
  - development relations
  - "proof" obligations
  - "proofs"
  - etc
- Editors (with syntax and type check)
- Translation (to Ada and C++)
- Pretty printing facilities (LaTeX)
- "Proof" tools





## Advantages of Using RAISE

- abstraction
- formal reasoning
- reuse
- tools support

=>

- fewer errors

## Example: RSL Specification (Model-oriented)

```
scheme SET_DATABASE =
  class
    type
      Database = Person-set,
      Person = Text

    value
      empty : Database  = {},

      register : Person × Database → Database
      register(p,db) ≡  db ∪ {p},

      check : Person × Database → Bool
      check(p,db) ≡  p ∈ db
  end
```





## Some immediate questions

- Do we need any more functions?
- Are the definitions correct?
- Are they what we need?
- Could we use this for registering the people in this class?
- Could we use this for registering the people in this country?
- Is **Text** a good model for Person?





## RSL Specification

An RSL specification consists of

- module definitions

A module contains definitions of

- types
- values
- variables
- channels
- modules
- axioms





## Abstraction

"What" rather than "how"

RSL allows

- data abstraction
- operation abstraction

## Data abstraction: examples

**type** Database = Person-**set**

**type** Database = Person*

**type** Database

**type** Person = **Text**

**type** Person = **Nat**

**type** Person





## Operation abstraction: example

**value**
square_root : **Real** $\xrightarrow{\sim}$ **Real**
square_root(r) **as** s
**post** s * s = r $\wedge$ s $\geq$ 0.0
**pre** r $\geq$ 0.0





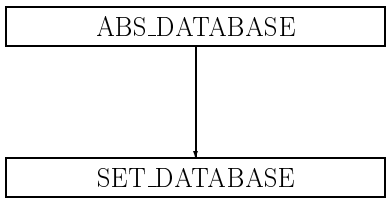
## Example: RSL Specification (Property-oriented)

**scheme** ABS_DATABASE =
**class**
**type**
Database,
Person
**value**
empty : Database,
register : Person $\times$ Database $\to$ Database,
check : Person $\times$ Database $\to$ **Bool**
**axiom**
$\forall$ p : Person $\cdot$
check(p, empty) $\equiv$ **false**,
$\forall$ p,p′ : Person, db : Database $\cdot$
check(p′, register(p, db)) $\equiv$ p′ = p $\vee$ check(p′, db)
**end**

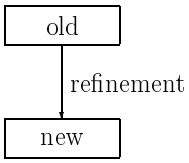




## A Development Step



## Refinement Relation




1. new signature includes the old
   (statically decidable)
2. old properties hold in the new
   (⇒ proof obligations: "refinement conditions")





## Refinement Conditions

⌞∀ p : Person · check(p, empty) ≡ **false**⌟

⌞∀ p,p′ : Person, db : Database ·
check(p′, register(p, db)) ≡ p′ = p ∨ check(p′, db)⌟





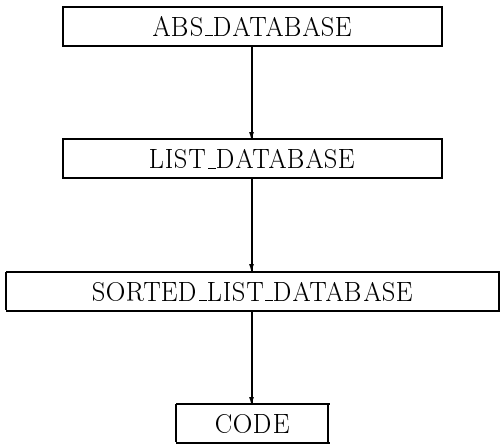
## Verification

⌞check(p′, register(p, db)) ≡ p′ = p ∨ check(p′, db)⌟
unfold register:
⌞check(p′, db ∪ {p}) ≡ p′ = p ∨ check(p′, db)⌟
unfold check:
⌞p′ ∈ (db ∪ {p}) ≡ p′ = p ∨ check(p′, db)⌟
unfold check:
⌞p′ ∈ (db ∪ {p}) ≡ p′ = p ∨ p′ ∈ db⌟
isin_union:
⌞p′ ∈ db ∨ p′ ∈ {p} ≡ p′ = p ∨ p′ ∈ db⌟
isin_singleton:
⌞p′ ∈ db ∨ p′ = p ≡ p′ = p ∨ p′ ∈ db⌟
or_commutativity:
⌞p′ = p ∨ p′ ∈ db ≡ p′ = p ∨ p′ ∈ db⌟
is_annihilation:
⌞**true**⌟
**qed**





## Alternative Development



## Summary

RAISE

- Method
  - stepwise development
  - invent and verify
  - rigorous
- Specification language (RSL)
  - formal
  - wide spectrum
  - structuring facilities
- Tools





RSL has a range of styles:

- applicative
- imperative
- concurrent

Most of the course will use the applicative style, which is close to mathematics and to functional programming.

The imperative style is closer to traditional programming, with program variables, assignments, sequencing and loops.

The concurrent style supports the specification of concurrent features of software.





# Syntax Overview





## Specifications

An RSL specification consists of

- module definitions

A module contains definitions of

- types
- values
- variables
- channels
- modules
- axioms

No special order of definitions is required.

# Module Definitions

id =
**class**
declaration$_1$
⋮
declaration$_n$
**end**





# Declarations

A declaration is a list of definitions of the same kind:

**type**
**value**
**axiom**
**variable**
**channel**
**scheme**
**object**

e.g.

**type**
type_definition$_1$,
⋮
type_definition$_n$

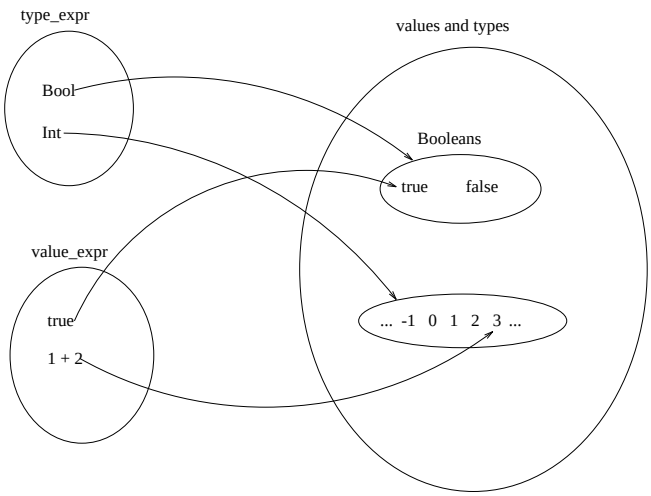
**value**
value_definition$_1$,
⋮
value_definition$_n$





# Types

Types are collections of values.








# Type Expressions

1. literals (**Int**, ...)
2. type operator applications (**Int-set**, ...)
3. subtype expressions ($\{|$ i : **Int** $\cdot$ i $\geq$ 0 $|\}$, ...)
4. identifiers defined in type definitions (Person)

All have associated $=$, $\neq$.

1 and 2:
have additional associated built-in operators ($+$, $\cup$,...).

3 and 4:
may have additional associated built-in operators.

## Type Definitions

- abbreviation
  **type** id = type_expr

  **type** Database = Person**-set**
- sort
  **type** id

  **type** Database





## Literals

**Bool**
values: **true**, **false**
connectives: $\wedge$, $\vee$, $\Rightarrow$, $\sim$

**Int**
values: ..., -2, -1, 0, 1, 2, ...
operators: $+$, $-$, $*$, $/$, $\uparrow$, $\backslash$, $<$, $\leq$, $>$, $\geq$, **abs**, **real**

**Nat** = $\{|\ \mathrm{i} : \textbf{Int} \cdot \mathrm{i} \geq 0\ |\}$

**Real**
values: ..., -4.3, ..., 0.0, ..., 1.0, ...
operators: $+$, $-$, $*$, $/$, $\uparrow$, $<$, $\leq$, $>$, $\geq$, **abs**, **int**

**Char**
values: $'\mathrm{a}'$, ...

**Text** = **Char**$^*$
values: $''\mathrm{Alice}''$, ...

**Unit**
value: ()





## Composite Types

- products ($\times$)
- functions ($\rightarrow$ , $\xrightarrow{\sim}$)
- sets (**-set**, **-infset**)
- lists ($^*$, $^\omega$)
- maps ( $\xrightarrow[\mathrm{m}]{}$, $\xrightarrow[\mathrm{m}]{\sim}$)





## Value Definitions

- different forms
  - typing (+ axiom(s))
  - explicit value definition
  - implicit value definition
  - explicit function definition
  - implicit function definition
- 4 last forms may be transformed to first form

## Constant Value Definitions

- explicit value definition

  **value** x : **Int** = 1

- implicit value definition

  **value** x : **Int** · x > 0

- typing (+ axiom(s))

  **value** x : **Int**
  **axiom** x > 0





## Function Value Definitions

- explicit function definition

  **value**
  f : **Int** → **Int**
  f(x) ≡ x + 1

- implicit function definition

  **value**
  f : **Int** → **Int**
  f(x) **as** r **post** r > x

- typing (+ axiom(s))

  **value** f : **Int** → **Int**
  **axiom** ∀ x : **Int** · f(x) > x





## Value expressions

Constructed from

- literals (1, **true**, ...)
- operators (+, ...)
- connectives (∧, ...)
- identifiers introduced in value definitions (empty, check, ...)
- function application (f(x))
- if expressions (**if** ...)
- quantified expressions (∀, ...)
- equivalence expressions
- ...





## Summary

- type definitions: abbreviation and sort
- type expressions
- value definitions: explicit, implicit and axiomatic
- value expressions

# Logic





Computing involves *partial* functions

- division
- head of a list
- loops

So we need a logic that can deal with expressions that may not be well defined.

By *well defined* we mean has (or evaluates to) a value.





## Expressions and values

An expression may or may not evaluate to a value:

| **Expression** | **Value** |
|---|---|
| **true** | **true** |
| $1 + 0$ | 1 |
| $1 \ / \ 0$ | ? |
| factorial(3) | 6 |
| factorial($-1$) | ? |
| factorial(x) | ? |
| **if** x $>$ 0 **then** factorial(x) **else** 0 **end** | $\surd$ |
| **while true do skip end** | $\times$ |





## chaos

Used to represent undefinedness

**while true do skip end** $\equiv$ **chaos**

$/ : \textbf{Real} \times \textbf{Real} \xrightarrow{\sim} \textbf{Real}$

1.0/0.0 is under-specified

1.0/0.0 might evaluate to **chaos**

f : **Real**→**Real**
f(x) $\equiv$ **if** x $\neq$ 0.0 **then** 1.0/x **else** 0.0 **end**

## If expressions

Example:

**if** x $>$ 0 **then** factorial(x) **else** 0 **end**

More general form:

**if** *logical*-expr **then** expr1 **else** expr2 **end**

Properties:

**if true then** expr1 **else** expr2 **end** $\equiv$ expr1
**if false then** expr1 **else** expr2 **end** $\equiv$ expr2
**if chaos then** expr1 **else** expr2 **end** $\equiv$ **chaos**

Non-strictness:

**if true then** expr1 **else chaos end** $\equiv$ expr1
**if false then chaos else** expr2 **end** $\equiv$ expr2





## Connectives

Properties:

$\sim$e $\equiv$ **if** e **then false else true end**

e1 $\wedge$ e2 $\equiv$ **if** e1 **then** e2 **else false end**

e1 $\vee$ e2 $\equiv$ **if** e1 **then true else** e2 **end**

e1 $\Rightarrow$ e2 $\equiv$ **if** e1 **then** e2 **else true end**

gives conditional logic





## Truth tables

| $\wedge$ | **true** | **false** | **chaos** |
|---|---|---|---|
| **true** | **true** | **false** | **chaos** |
| **false** | **false** | **false** | **false** |
| **chaos** | **chaos** | **chaos** | **chaos** |

| $\vee$ | **true** | **false** | **chaos** |
|---|---|---|---|
| **true** | **true** | **true** | **true** |
| **false** | **true** | **false** | **chaos** |
| **chaos** | **chaos** | **chaos** | **chaos** |

| $\Rightarrow$ | **true** | **false** | **chaos** |
|---|---|---|---|
| **true** | **true** | **false** | **chaos** |
| **false** | **true** | **true** | **true** |
| **chaos** | **chaos** | **chaos** | **chaos** |





Note:

e1 $\wedge$ e2 $\equiv$ e2 $\wedge$ e1
e1 $\vee$ e2 $\equiv$ e2 $\vee$ e1

are not tautologies

p : **Real** $\rightarrow$ **Bool**
p(x) $\equiv$ (x $\neq$ 0.0) $\wedge$ (1.0/x $\leq$ epsilon)

p(0.0)
$\equiv$ (0.0 $\neq$ 0.0) $\wedge$ (1.0/0.0 $\leq$ epsilon)
$\equiv$ **if** (0.0 $\neq$ 0.0) **then** (1.0/0.0 $\leq$ epsilon) **else false end**
$\equiv$ **if false then** (1.0/0.0 $\leq$ epsilon) **else false end**
$\equiv$ **false**

## Quantified expressions

Examples:

$\forall$ x : **Nat** $\cdot$ (x = 0) $\vee$ (x > 0)

$\exists$ x : **Int** $\cdot$ x = 7

$\exists!$ x : **Int** $\cdot$ (x $\geq$ 0) $\wedge$ (x $\leq$ 0)

$\forall$ x : **Nat** $\cdot$ x = -7

$\forall$ x, y : **Nat** $\cdot$ ($\exists!$ z : **Nat** $\cdot$ x+y = z)

General form:

quantifier typing$_1$, ... , typing$_n$ $\cdot$ *logical*-expr

All quantification is over values in the types stated,
i.e. not over **chaos**.





# Products





## Products

A product is:

an ordered finite collection of values of possibly different types

Examples:

(1,2)
(1,**true**,"`John`")





## Product Type Expressions

type_expr$_1$ $\times$ ... $\times$ type_expr$_n$, n $\geq$ 2

Values:

($v_1$,...,$v_n$), $v_i$ : type_expr$_i$

Operators:

=
$\neq$

## Examples

**Bool × Bool**

(**true**,**true**)
(**true**,**false**)
(**false**,**true**)
(**false**,**false**)

**Nat × Nat × Bool**

(0,0,**true**)
(0,0,**false**)
(0,1,**true**)
(0,1,**false**)
(1,0,**true**)
(1,0,**false**)
(2,0,**true**)
⋮





## Example: A System of Coordinates I

SYSTEM_OF_COORDINATES =
**class**
**type**
Position = **Real** × **Real**
**value**
origin : Position = (0.0,0.0),
distance : Position × Position → **Real**
distance((x$_1$,y$_1$),(x$_2$,y$_2$)) ≡
((x$_2$−x$_1$)↑2.0 + (y$_2$−y$_1$)↑2.0)↑0.5
**end**





## Example: A System of Coordinates II

SYSTEM_OF_COORDINATES =
**class**
**type**
Position = **Real** × **Real**
**value**
origin : Position = (0.0,0.0),
distance : Position × Position → **Real**
distance(p1,p2) ≡
**let**
(x$_1$,y$_1$) = p1,
(x$_2$,y$_2$) = p2
**in**
((x$_2$−x$_1$)↑2.0 + (y$_2$−y$_1$)↑2.0)↑0.5
**end**
**end**





## (Explicit) Let Expressions

**let**
binding$_1$ = expr$_1$,
⋮
binding$_n$ = expr$_n$
**in**
expr
**end**

## Bindings

Examples:

x
(x,y)
(x,y,z)
((x1,y1),(x2,y2))
(x,(y,z))

Forms:

id
($\text{binding}_1$, ... , $\text{binding}_n$)

Use:

- typings
- let expressions
- formal function parameters
- ...





## Typings

Examples:

x : **Real**
(x,y) : Position
a, b : **Real**

Basic Form:

binding : type_expr

Context condition:

binding must match type_expr

Derived form:

$\text{binding}_1$, ... , $\text{binding}_n$ : type_expr

Use:

- value definitions
- quantified expressions
- ...





# Functions





## Total and Partial Functions

- total functions

  $\text{type_expr}_1 \to \text{type_expr}_2$

- partial functions

  $\text{type_expr}_1 \xrightarrow{\sim} \text{type_expr}_2$

$\forall\ f_{tot} : T_1 \to T_2,\ f_{par} :\ T_1 \xrightarrow{\sim} T_2,\ x : T_1 \cdot$

| | defined (not **chaos**) | deterministic |
|---|---|---|
| $f_{tot}(x)$ | yes | yes |
| $f_{par}(x)$ | might be | might be |

$\exists!\ y :\ T_2 \cdot f_{tot}(x) \equiv y$

## Preconditions

Partial functions are usually defined with preconditions.

For example:

factorial : **Int** $\xrightarrow{\sim}$ **Int**
factorial(x) ≡
  **if** x = 1 **then** 1 **else** x ∗ factorial(x − 1) **end**
**pre** x > 0

The defining expression for the function can only be used when the precondition is true. So we can deduce that

f(3) = 3 ∗ f(2) = ... = 3 ∗ 2 ∗ 1 = 6

but we can deduce nothing about f(0), say.





## Function Value Expressions

Basic form:

λ binding : type_expr · value_expr

Examples:

λ b : **Bool** · ∼b
λ (x,y) : **Int** × **Int** · x + y
λ (b,(x,y)) : **Bool** × (**Nat** × **Nat**) ·
  **if** b **then** x **else** y **end**

Semantics:

represents function of type: type_expr $\xrightarrow{\sim}$ T,
where T = type_of(value_expr)





## Function Application Expressions

Examples:

fraction(0.5)
**abs**(−7)

Typical form:

*function*-expr($\text{expr}_1$, ... , $\text{expr}_n$), n ≥ 0

Context conditions:

($\text{expr}_1$, ... , $\text{expr}_n$)

must be of the argument type of expr





## Associated Built-in Operators

=, ≠, ∘

∘ : ($T_2 \xrightarrow{\sim} T_3$) × ($T_1 \xrightarrow{\sim} T_2$) → ($T_1 \xrightarrow{\sim} T_3$)
(f ∘ g)(x) ≡ f(g(x))

## Function Value Definitions

- explicit function definition

  **value**
  f : **Int** → **Int**
  f(x) ≡ x + 1

- implicit function definition

  **value**
  f : **Int** → **Int**
  f(x) **as** r **post** r > x

- typing (+ axiom(s))

  **value** f : **Int** → **Int**
  **axiom** ∀ x : **Int** · f(x) > x





## Explicit Function Definition

**value**
fraction : **Real** → **Real**
fraction(x) ≡ **if** x = 0.0 **then** 0.0 **else** 1.0/x **end**

short for

**value**
fraction : **Real** → **Real**
**axiom**
∀ x : **Real** ·
fraction(x) ≡ **if** x = 0.0 **then** 0.0 **else** 1.0/x **end**





## Explicit Function Definition

**value**
partial_fraction : **Real** $\xrightarrow{\sim}$ **Real**
partial_fraction(x) ≡ 1.0/x
**pre** x ≠ 0.0

short for

**value**
partial_fraction : **Real** $\xrightarrow{\sim}$ **Real**
**axiom**
∀ x : **Real** ·
partial_fraction(x) ≡ 1.0/x
**pre** x ≠ 0.0





## Implicit Function Definition

**value**
square_root : **Real** $\xrightarrow{\sim}$ **Real**
square_root(x) **as** s
**post** s ∗ s = x ∧ s ≥ 0.0
**pre** x ≥ 0.0

short for:

**value**
square_root : **Real** $\xrightarrow{\sim}$ **Real**
**axiom**
∀ x : **Real** ·
square_root(x) **as** s
**post** s ∗ s = x ∧ s ≥ 0.0
**pre** x ≥ 0.0

## Algebraic Specification Example

DATABASE =
**class**
**type**
Database, Key, Data
**value**
/∗ generators ∗/
empty : Database,
insert : Key × Data × Database → Database,
remove : Key × Database → Database,
/∗ observers ∗/
defined : Key × Database → **Bool**,
lookup : Key × Database $\xrightarrow{\sim}$ Data





## DATABASE contd.

**axiom**
[ defined_empty ]
∀ k : Key ·
defined(k,empty) ≡ **false**,
[ defined_insert ]
∀ k,k1 : Key, d : Data, db : Database ·
defined(k,insert(k1,d,db)) ≡
k = k1 ∨ defined(k,db),
[ defined_remove ]
∀ k,k1 : Key, db : Database ·
defined(k,remove(k1,db)) ≡
k ≠ k1 ∧ defined(k,db),
[ lookup_insert ]
∀ k,k1 : Key, d : Data, db : Database ·
lookup(k,insert(k1,d,db)) ≡
**if** k = k1 **then** d **else** lookup(k,db) **end**
**pre** k = k1 ∨ defined(k,db),
[ lookup_remove ]
∀ k,k1 : Key, db : Database ·
lookup(k,remove(k1,db)) ≡ lookup(k,db)
**pre** k ≠ k1 ∧ defined(k,db)
**end**





## Function Definition

1. Decide name (f)
2. Decide type
   (a) Argument type $T_a$
   (b) Result type $T_r$
   (c) Total ($\rightarrow$) or partial ($\xrightarrow{\sim}$):
      i. total: if can define for all values in parameters
      ii. partial: if a pre condition is necessary
3. Decide definition style:
   (a) explicit:
   if it is possible to state a formula
   f(x) ≡ expr[x]
   (b) implicit:
   if it is possible to write an input-output relation
   p[x,r]
   (c) axiomatic:
   - always possible
   - typically used in connection with sorts
   - necessary, if special argument forms are wanted
   e.g. f(g(x),x) ≡ h(f,x)





## Sets

## Contents

Sets

- finite and infinite sets
- set type expressions
- set operators
- set value expressions
- examples of abstraction using sets





## Sets

A set is:

an unordered collection of values of same type

Examples:

{1,3,5}
{"John","Peter","Ann"}





## Set Type Expressions

- type_expr**-set**

  $\{v_1, \ldots, v_n\}$

  where $n \geq 0$, $v_i$ : type_expr

- type_expr**-infset**

  $\{v_1, \ldots, v_n\}$,
  $\{v_1, \ldots, v_n, \ldots\}$

  where $n \geq 0$, $v_i$ : type_expr





## Examples

### Bool-set

{}
{**true**}
{**false**}
{**true**,**false**}

### Nat-set

{}
{0}
{1}
...
{0,1}
...
{1,2,3}
...

## Associated Built-in Operators

$\cup$ : T-**infset** $\times$ T-**infset** $\to$ T-**infset**

$\{1, 2, 3\} \cup \{3, 4\} = \{1, 2, 3, 4\}$

$\cap$ : T-**infset** $\times$ T-**infset** $\to$ T-**infset**

$\{1, 2, 3\} \cap \{3, 4\} = \{3\}$

$\setminus$ : T-**infset** $\times$ T-**infset** $\to$ T-**infset**

$\{1, 2, 3\} \setminus \{3, 4\} = \{1, 2\}$

$\in$ : T $\times$ T-**infset** $\to$ **Bool**

$4 \in \{1, 2, 3\} =$ **false**

$\notin$ : T $\times$ T-**infset** $\to$ **Bool**

$4 \notin \{1, 2, 3\} =$ **true**





$\subset$ : T-**infset** $\times$ T-**infset** $\to$ **Bool**

$\{1, 3\} \subset \{1, 2, 3\} =$ **true**
$\{1, 2, 3\} \subset \{1, 2, 3\} =$ **false**

$\subseteq$ : T-**infset** $\times$ T-**infset** $\to$ **Bool**

$\{1, 3\} \subseteq \{1, 2, 3\} =$ **true**
$\{1, 2, 3\} \subseteq \{1, 2, 3\} =$ **true**
$\{1, 2, 3\} \subseteq \{1, 3\} =$ **false**

$\supset$ and $\supseteq$ are similar

**card** : T-**infset** $\xrightarrow{\sim}$ **Nat**

**card** $\{1, 2, 5, 2, 2, 1, 5\} = 3$
**card** $\{n \mid n : \textbf{Nat}\} \equiv$ **chaos**





## Set Value Expressions

Enumerated:

$\{1,2\}$
$\{1,2,1\}$

$\{expr_1,...,expr_n\}$

Ranged:

$\{3 .. 7\} = \{3,4,5,6,7\}$
$\{3 .. 3\} = \{3\}$
$\{3 .. 2\} = \{\}$

$\{$*integer*-expr$_1$ .. *integer*-expr$_2\}$

Comprehended:

$\{2{*}n \mid n : \textbf{Nat} \cdot n \leq 3\}$

$\{expr_1 \mid typing_1,...,typing_n \cdot$ *logical*-expr$_2\}$





RESOURCE_MANAGER =
**class**
**type**
Resource,
Pool = Resource-**set**

**value**
obtain : Pool $\xrightarrow{\sim}$ Pool $\times$ Resource
obtain(p) **as** ($p_1$,$r_1$) **post** $r_1 \in p \wedge p_1 = p \setminus \{r_1\}$
**pre** $p \neq \{\}$,

release : Resource $\times$ Pool $\xrightarrow{\sim}$ Pool
release(r,p) $\equiv p \cup \{r\}$
**pre** $r \notin p$
**end**

SET_DATABASE =
**class**
**type**
Record = Key × Data,
Database = {| rs : Record-**set** · is_wf_Database(rs) |},
Key, Data
**value**
is_wf_Database : Record-**set** → **Bool**
is_wf_Database(rs) ≡
(∀ k : Key, d1,d2 : Data ·
((k,d1) ∈ rs ∧ (k,d2) ∈ rs) ⇒ d1 = d2),

empty : Database = {},

insert : Key × Data × Database → Database
insert(k,d,db) ≡ remove(k,db) ∪ {(k,d)},

remove : Key × Database → Database
remove(k,db) ≡ db \ {(k,d) | d : Data · **true**},

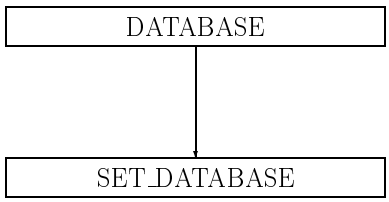
defined : Key × Database → **Bool**
defined(k,db) ≡ (∃ d : Data · (k,d) ∈ db),

lookup : Key × Database ~→ Data
lookup(k,db) **as** d **post** (k,d) ∈ db
**pre** defined(k,db)
**end**





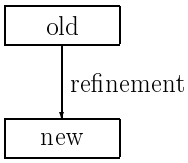
## A Development Step








## Refinement Relation




1. new signature includes the old
(statically decidable)
2. old properties hold in the new
(⇒ "refinement conditions")





## Refinement Conditions

⌞∀ k : Key · defined(k,empty) ≡ **false**⌟

⌞∀ k,k1 : Key, d : Data, db : Database ·
defined(k,insert(k1,d,db)) ≡
k = k1 ∨ defined(k,db)⌟

⌞∀ k,k1 : Key, db : Database ·
defined(k,remove(k1,db)) ≡
k ≠ k1 ∧ defined(k,db)⌟

⌞∀ k,k1 : Key, d : Data, db : Database ·
lookup(k,insert(k1,d,db)) ≡
**if** k = k1 **then** d **else** lookup(k,db) **end**
**pre** k = k1 ∨ defined(k,db)⌟

⌞∀ k,k1 : Key, db : Database ·
lookup(k,remove(k1,db)) ≡ lookup(k,db)
**pre** k ≠ k1 ∧ defined(k,db)⌟

## Verification of a Refinement Condition

⌞∀ k : Key · defined(k,empty) ≡ **false**⌟
all_assumption_inf:
⌞defined(k,empty) ≡ **false**⌟
unfold empty:
⌞defined(k,{}) ≡ **false**⌟
unfold defined:
⌞(∃ d : Data · (k,d) ∈ {}) ≡ **false**⌟
isin_empty:
⌞(∃ d : Data · **false**) ≡ **false**⌟
exists_introduction:
⌞**false** ≡ **false**⌟
is_annihilation:
⌞**true**⌟
**qed**





## Lists





## Contents

Lists

- finite and infinite lists
- list type expressions
- list value expressions
- list indexing
- list operators
- examples of abstraction using lists





## Lists

A list is:

an ordered collection of values of same type

Examples:

⟨1,3,3,1,5⟩
⟨**true**,**false**,**true**⟩

# List Type Expressions

- type_expr$^*$

  $\langle v_1, ..., v_n \rangle$

  where $n \geq 0$, $v_i$ : type_expr

- type_expr$^\omega$

  $\langle v_1, ..., v_n \rangle$,
  $\langle v_1, ..., v_n, ... \rangle$

  where $n \geq 0$, $v_i$ : type_expr





# Examples

**Bool**$^*$

$\langle\rangle$
$\langle$**true**$\rangle$
$\langle$**false**$\rangle$
$\langle$**true**,**false**$\rangle$
$\langle$**false**,**true**$\rangle$
$\langle$**true**,**true**$\rangle$
$\langle$**false**,**false**$\rangle$
$\langle$**true**,**false**,**true**$\rangle$
⋮

**Bool**$^\omega$

$\langle\rangle$
$\langle$**true**$\rangle$
$\langle$**false**$\rangle$
$\langle$**true**,**false**$\rangle$
$\langle$**false**,**true**$\rangle$
$\langle$**true**,**true**$\rangle$
$\langle$**false**,**false**$\rangle$
$\langle$**true**,**false**,**true**$\rangle$
⋮
$\langle$**false**,**true**,**true**,**true**,**false**,... $\rangle$
⋮





# List Value Expressions

Enumerated:

$\langle$1,3,3,1,5$\rangle$
$\langle$**true**,**false**,**true**$\rangle$

$\langle expr_1,...,expr_n \rangle$

Ranged:

$\langle 3 .. 7 \rangle = \langle 3,4,5,6,7 \rangle$
$\langle 3 .. 3 \rangle = \langle 3 \rangle$
$\langle 3 .. 2 \rangle = \langle\rangle$

$\langle$*integer*-expr$_1$ .. *integer*-expr$_2\rangle$

Comprehended:

$\langle 2*n \mid n$ **in** $\langle 0 .. 3 \rangle \rangle$
$\langle n \mid n$ **in** $\langle 0 .. 100 \rangle \cdot$ is_even(n)$\rangle$

$\langle expr_1 \mid$ binding **in** *list*-expr$_2 \cdot$ *logical*-expr$_3\rangle$





# List Indexing

Basic form:

*list*-expr(*integer*-expr$_1$)

Example:

$\langle 2,5,3 \rangle(2) = 5$

Derived form:

*list*-expr(*integer*-expr$_1$)...(*integer*-expr$_n$)

Example:

$\langle\langle 2,5,3 \rangle, \langle 3 \rangle\rangle(1) = \langle 2,5,3 \rangle$
$\langle\langle 2,5,3 \rangle, \langle 3 \rangle\rangle(1)(2) = \langle 2,5,3 \rangle(2) = 5$

## Associated Built-in Operators

$\frown : T^* \times T^\omega \to T^\omega$

$\langle e_1,...,e_n \rangle \frown \langle e_{n+1},... \rangle = \langle e_1,...,e_n,e_{n+1},... \rangle$

**hd** : $T^\omega \xrightarrow{\sim} T$

**hd** $\langle e_1,e_2,... \rangle = e_1$

**tl** : $T^\omega \xrightarrow{\sim} T^\omega$

**tl** $\langle e_1,e_2,... \rangle = \langle e_2,... \rangle$

**len** : $T^\omega \xrightarrow{\sim}$ **Nat**

**len** $\langle e_1,...,e_n \rangle = n$
**len** il $\equiv$ **chaos**

**elems** : $T^\omega \to T$**-infset**

**elems** $\langle e_1,e_2,... \rangle = \{e_1,e_2,... \}$

**inds** : $T^\omega \to$ **Nat-infset**

**inds** fl $= \{1$ .. **len** fl$\}$
**inds** il $= \{$idx | idx : **Nat** $\cdot$ idx $\geq 1\}$





```
QUEUE =
  class
    type
      Element,
      Queue = Element*
    value
      empty : Queue = ⟨⟩,

      enq : Element × Queue → Queue
      enq(e,q) ≡ q ⌢ ⟨e⟩,

      deq : Queue ~→ Queue × Element
      deq(q) ≡ (tl q,hd q)
      pre q ≠ empty
  end
```



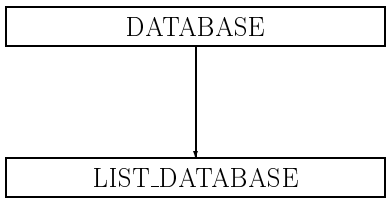


```
SORTING =
  class
    value
      sort : Int* → Int*
      sort(l) as l1 post is_permutation(l1,l) ∧ is_sorted(l1)

      is_permutation : Int* × Int* → Bool,
      is_permutation(l1,l2) ≡
        (∀ i : Int · count(i, l1) = count(i, l2)),

      count : Int × Int* → Nat
      count(i, l) ≡
        card {idx | idx : Nat ·
                  idx ∈ inds l ∧ l(idx) = i},

      is_sorted : Int* → Bool
      is_sorted(l) ≡
        (∀ idx1,idx2 : Nat ·
          {idx1,idx2} ⊆ inds l ∧ idx1 < idx2 ⇒
            l(idx1) ≤ l(idx2))
  end
```





## A Development Step



# Database Representations

**type** Database

empty
insert($k_1$, $d_1$, insert($k_2$,$d_2$,empty))

**type** Database = (Key × Data)**-set**

{}
{($k_1$,$d_1$), ($k_2$,$d_2$)}

**type** Database = (Key × Data)*

⟨⟩
⟨($k_1$,$d_1$), ($k_2$,$d_2$)⟩





```
LIST_DATABASE =
  class
    type
      Key, Data,
      Record = Key × Data,
      Database = Record*
    value
      empty : Database = ⟨⟩,

      insert : Key × Data × Database → Database
      insert(k,d,db) ≡ ⟨(k,d)⟩ ⌢ db,

      remove : Key × Database → Database
      remove(k,db) ≡
        case db of
          ⟨⟩ → ⟨⟩,
          ⟨(k1,d1)⟩ ⌢ db1 →
            if k = k1 then remove(k,db1)
            else ⟨(k1,d1)⟩ ⌢ remove(k,db1) end
        end,
```





```
      defined : Key × Database → Bool
      defined(k,db) ≡
        case db of
          ⟨⟩ → false,
          ⟨(k1,d1)⟩ ⌢ db1 → k = k1 ∨ defined(k,db1)
        end,

      lookup : Key × Database ~→ Data
      lookup(k, db) ≡
        let (k1,d1) = hd db in
          if k = k1 then d1
          else lookup(k, tl db) end
        end
      pre defined(k, db)
  end
```





```
    ⌞∀ k : Key · remove(k,empty) ≡ empty⌟
  all_assumption_inf:
    ⌞remove(k,empty) ≡ empty⌟
  unfold empty, unfold empty:
    ⌞remove(k,⟨⟩) ≡ ⟨⟩⌟
  unfold remove:
    ⌞case ⟨⟩ of ⟨⟩ → ⟨⟩, ... end ≡ ⟨⟩⌟
  case_expansion:
    ⌞⟨⟩ ≡ ⟨⟩⌟
  is_annihilation:
    ⌞true⌟
  qed
```

# Maps





## Contents

Maps

- map type expressions
- map value expressions
- map application
- map operators
- examples of abstraction using maps





## Maps

A map is:

an unordered collection of pairs of values

Examples:

$[\text{"}\texttt{Klaus}\text{"} \mapsto 7, \text{"}\texttt{John}\text{"} \mapsto 2, \text{"}\texttt{Mary}\text{"} \mapsto 7]$
$[1 \mapsto 2, 5 \mapsto 10]$





Maps may be:

- infinite
- partial
- non-deterministic

# Map Type Expressions

- $\text{type_expr}_1 \xrightarrow[\text{m}]{} \text{type_expr}_2$

  $[\text{v}_1 \mapsto \text{w}_1 , \ldots , \text{v}_\text{n} \mapsto \text{w}_\text{n}]$

  where $\text{n} \geq 0$, $\text{v}_\text{i} : \text{type_expr}_1$, $\text{w}_\text{i} : \text{type_expr}_2$
  and $\text{v}_\text{i} = \text{v}_\text{j} \Rightarrow \text{w}_\text{i} = \text{w}_\text{j}$

  Finite and deterministic when applied to elements in the domain

- $\text{type_expr}_1 \xrightarrow[\text{m}]{\sim} \text{type_expr}_2$

  $[\text{v}_1 \mapsto \text{w}_1 , \ldots , \text{v}_\text{n} \mapsto \text{w}_\text{n}]$,
  $[\text{v}_1 \mapsto \text{w}_1 , \ldots , \text{v}_\text{n} \mapsto \text{w}_\text{n}, \ldots]$,

  where $\text{n} \geq 0$, $\text{v}_\text{i} : \text{type_expr}_1$, $\text{w}_\text{i} : \text{type_expr}_2$

  May be infinite and may be non-deterministic when applied to elements in the domain

**NB** The original RSL book only has $\xrightarrow[\text{m}]{}$, but with the meaning of $\xrightarrow[\text{m}]{\sim}$ Finite maps were introduced and the symbols changed in the method book.





# Example

**Nat $\xrightarrow[\text{m}]{}$ Bool**

$[\,]$
$[0 \mapsto \textbf{true}]$
$[0 \mapsto \textbf{true}, 1 \mapsto \textbf{true}]$
$[0 \mapsto \textbf{true}, 1 \mapsto \textbf{false}]$
$\vdots$

**Nat $\xrightarrow[\text{m}]{\sim}$Bool**

$[\,]$
$[0 \mapsto \textbf{true}]$
$[0 \mapsto \textbf{true}, 1 \mapsto \textbf{true}]$
$[0 \mapsto \textbf{true}, 1 \mapsto \textbf{false}]$
$[0 \mapsto \textbf{true}, 0 \mapsto \textbf{false}]$
$[0 \mapsto \textbf{true}, 0 \mapsto \textbf{false}, 1 \mapsto \textbf{true}]$
$\vdots$





# Map Value Expressions

Enumerated:

$[3 \mapsto \textbf{true}, 5 \mapsto \textbf{false}]$
$[\text{"}\texttt{Klaus}\text{"} \mapsto 7, \text{"}\texttt{John}\text{"} \mapsto 2, \text{"}\texttt{Mary}\text{"} \mapsto 7]$

$[\text{expr}_1 \mapsto \text{expr}_1', \ldots, \text{expr}_\text{n} \mapsto \text{expr}_\text{n}']$

Comprehended:

$[\text{n} \mapsto 2{*}\text{n} \mid \text{n} : \textbf{Nat} \cdot \text{n} \leq 2] = [0 \mapsto 0, 1 \mapsto 2, 2 \mapsto 4]$
$[\text{n} \mapsto 2{*}\text{n} \mid \text{n} : \textbf{Nat}] = [0 \mapsto 0, 1 \mapsto 2, 2 \mapsto 4, \ldots]$

$[\text{expr}_1 \mapsto \text{expr}_2 \mid \text{typing}_1, \ldots, \text{typing}_\text{n} \cdot \textit{logical}\text{-expr}_3]$





# Map Application

Basic form:

$\textit{map}\text{-expr}(\text{expr}_1)$

Examples:

$[\text{"}\texttt{Klaus}\text{"} \mapsto 7, \text{"}\texttt{John}\text{"} \mapsto 2, \text{"}\texttt{Mary}\text{"} \mapsto 7](\text{"}\texttt{John}\text{"}) = 2$

$[3 \mapsto \textbf{true}, 3 \mapsto \textbf{false}](3) = \textbf{true} \sqcap \textbf{false}$

Derived form:

$\textit{map}\text{-expr}(\text{expr}_1)\ldots(\text{expr}_\text{n})$

Example:

$[1 \mapsto [\text{"}\texttt{Per}\text{"} \mapsto 5, \text{"}\texttt{Jan}\text{"} \mapsto 7] , 2 \mapsto [\,]](1)(\text{"}\texttt{Jan}\text{"})$

## Associated Built-in Operators

**dom** : $(T_1 \xrightarrow[m]{\sim} T_2) \to T_1$**-infset**

**dom** $[3 \mapsto$ **true**, $5 \mapsto$ **false**$] = \{3, 5\}$
**dom** $[3 \mapsto$ **true**, $5 \mapsto$ **false**, $5 \mapsto$ **true**$] = \{3, 5\}$

**rng** : $(T_1 \xrightarrow[m]{\sim} T_2) \to T_2$**-infset**

**rng** $[3 \mapsto$ **false**, $5 \mapsto$ **false**$] =$ **{false}**
**rng** $[3 \mapsto$ **false**, $5 \mapsto$ **false**, $5 \mapsto$ **true**$] =$ **{false, true}**

$\dagger : (T_1 \xrightarrow[m]{\sim} T_2) \times (T_1 \xrightarrow[m]{\sim} T_2) \to (T_1 \xrightarrow[m]{\sim} T_2)$

$[3 \mapsto$ **true**, $5 \mapsto$ **false**$] \dagger [5 \mapsto$ **true**$]$
$= [3 \mapsto$ **true**, $5 \mapsto$ **true**$]$

$\cup : (T_1 \xrightarrow[m]{\sim} T_2) \times (T_1 \xrightarrow[m]{\sim} T_2) \to (T_1 \xrightarrow[m]{\sim} T_2)$

$[3 \mapsto$ **true**, $5 \mapsto$ **false**$] \cup [5 \mapsto$ **true**$]$
$= [3 \mapsto$ **true**, $5 \mapsto$ **false**, $5 \mapsto$ **true**$]$





$\setminus : (T_1 \xrightarrow[m]{\sim} T_2) \times T_1$**-infset** $\to (T_1 \xrightarrow[m]{\sim} T_2)$

$m \setminus s = [d \mapsto m(d) \mid d : T_1 \cdot d \in$ **dom** $m \wedge d \notin s]$
$[3 \mapsto$ **true**, $5 \mapsto$ **false**$] \setminus \{5, 7\} = [3 \mapsto$ **true**$]$

$/ : (T_1 \xrightarrow[m]{\sim} T_2) \times T_1$**-infset** $\to (T_1 \xrightarrow[m]{\sim} T_2)$

$m / s = [d \mapsto m(d) \mid d : T_1 \cdot d \in$ **dom** $m \wedge d \in s]$
$[3 \mapsto$ **true**, $5 \mapsto$ **false**$] / \{5, 7\} = [5 \mapsto$ **false**$]$

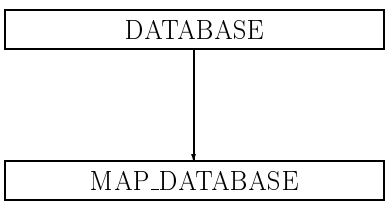
$\circ : (T_2 \xrightarrow[m]{\sim} T_3) \times (T_1 \xrightarrow[m]{\sim} T_2) \to (T_1 \xrightarrow[m]{\sim} T_3)$

$m_1 \circ m_2 =$
$[x \mapsto m_1(m_2(x)) \mid x : T_1 \cdot$
$x \in$ **dom** $m_2 \wedge m_2(x) \in$ **dom** $m_1]$
$[3 \mapsto$ **true**, $5 \mapsto$ **false**$] \circ [''$`Klaus`$'' \mapsto 3, ''$`John`$'' \mapsto 7]$
$= [''$`Klaus`$'' \mapsto$ **true**$]$





## A Development Step








## Database Representations

**type** Database

empty
insert($k_1$, $d_1$, insert($k_2$,$d_2$,empty))

**type** Database = (Key $\times$ Data)**-set**

$\{\}$
$\{(k_1,d_1), (k_2,d_2)\}$

**type** Database = (Key $\times$ Data)$^*$

$\langle\rangle$
$\langle(k_1,d_1), (k_2,d_2)\rangle$

**type** Database = Key $\xrightarrow[m]{}$ Data

$[\,]$
$[k_1 \mapsto d_1, k_2 \mapsto d_2]$

```
MAP_DATABASE =
  class
    type
      Database = Key ~m~> Data,
      Key, Data
    value
      empty : Database = [ ],

      insert : Key × Data × Database → Database
      insert(k,d,db) ≡ db † [ k ↦ d ],

      remove : Key × Database → Database
      remove(k,db) ≡ db \ {k},

      defined : Key × Database → Bool
      defined(k,db) ≡ k ∈ dom db,

      lookup : Key × Database ~~> Data
      lookup(k,db) ≡ db(k)
      pre defined(k,db)
  end
```





# Subtypes





## Contents

Subtypes

- subtype expressions
- maximal types and type checking





## Subtype Expressions

Examples:

{| l : **Int**$^*$ · **len** l > 0 |}
{| rs : Record-**set** · is_wf_Database(rs) |}

General form:

{| binding : type_expr · *logical*-value_expr |}

## Maximal types

The maximal types are

- **Bool**, **Int**, **Real**, **Char**, **Unit**
- Sorts
- Type expressions composed from maximal types and the type constructors $\times$, **-infset**, $^{\omega}$, $\xrightarrow[m]{\sim}$ $\xrightarrow{\sim}$
- Type identifiers defined as abbreviations for maximal types.

Examples of non-maximal types:

- **Nat**, **Text** (= **Char**$^*$)
- Type expressions involving the type constructors **-set**, $^*$, $\xrightarrow[m]{}$, $\rightarrow$
- Subtypes (unless the type_expr is maximal and the *logical*-value_expr is **true**)

Type checking only checks maximal types.





## Example

If f is defined

**value**
  f : **Nat** $\rightarrow$ **Int**
  f(n) $\equiv$
    **if** n = 0 $\vee$ n = 1 **then** 1
    **else** n $*$ f(n$-$1) **end**

then f($-$1) is not a type error.





## Type Definitions





## Type Definitions

- abbreviation

  **type** id = type_expr

- sort

  **type** id

- variant

  **type** id == variant$_1$ | ... | variant$_n$

  where n $\geq$ 1

- record

  **type** id ::
    $d_1$ : type_expr$_1$
    $\vdots$
    $d_n$ : type_expr$_n$

  where n $\geq$ 1

- union

  **type** id = id$_1$ | ... | id$_n$

  where n $\geq$ 2

**type**
  type_definition$_1$,
  ⋮
  type_definition$_n$

Any order of definitions is allowed.

Cyclic abbreviation definitions are not allowed.





## Name versus Structural Equivalence

Abbreviation definitions: structural equivalence

Other type definitions: name equivalence

**type**
  A = **Int**, B = **Int**
**value**
  a : A, b : B
**axiom**
  /∗ correct ∗/ a = b

**type**
  A , B
**value**
  a : A, b : B
**axiom**
  /∗ incorrect ∗/ a = b

**type**
  A :: **Int**, B :: **Int**
**value**
  a : A, b : B
**axiom**
  /∗ incorrect ∗/ a = b,
  /∗ incorrect ∗/ mk_A(2) = mk_B(2)





## Record Definitions

**type**
  Book ::
    title : Title
    author : Author
    publisher : Publisher
    year : Year
    price : Price ↔ new_price

title : Book → Title
⋮
price : Book → Price  are *destructors*

new_price : Price × Book → Book  is a *reconstructor*

mk_Book :
  Title × Author × Publisher × Year × Price → Book
is the *constructor*





## Record Definitions

**type**
  id ::
    $d_1$ : type_expr$_1$ ↔ $r_1$
    ⋮
    $d_n$ : type_expr$_n$ ↔ $r_n$

Values:

  mk_id($v_1$, ... , $v_n$)  where $v_i$ : type_expr$_i$

Associated functions:

  mk_id : type_expr$_1$ × ... × type_expr$_n$ → id

  $d_1$ : id → type_expr$_1$
  ⋮
  $d_n$ : id → type_expr$_n$

  $r_1$ : type_expr$_1$ × id → id
  ⋮
  $r_n$ : type_expr$_n$ × id → id

## Records versus Products

1. **type** id :: $d_1$ : type_expr$_1$ ... $d_n$ : type_expr$_n$

2. **type** id = type_expr$_1$ × ... × type_expr$_n$

Use record (1) when

- destructor functions will be useful
- id should be distinct from type_expr$_1$ × ... × type_expr$_n$





**type**
  Book ::
    title : Title
    price : Price ↔ new_price

is short for

**type** Book
**value**
  mk_Book : Title × Price → Book,
  title : Book → Title
  price : Book → Price
  new_price : Price × Book → Book
**axiom**
  [title_mk_Book]
    ∀ t : Title, p : Price ·
      title(mk_Book(t, p)) ≡ t,
  [price_mk_Book]
    ∀ t : Title, p : Price ·
      price(mk_Book(t, p)) ≡ p,
  [new_price_mk_Book]
    ∀ t : Title, p, p′ : Price ·
      new_price(p′, mk_Book(t, p)) ≡ mk_Book(t, p′),
  [Book_induction]
    ∀ pred : Book → **Bool** ·
      (∀ t : Title, p : Price · pred(mk_Book(t, p))) ⇒
        (∀ b : Book · pred(b))





## Variant definitions 1

COLOUR =
  **class**
    **type**
      Colour == red | green
    **value**
      is_primary : Colour → **Bool**
      is_primary(c) ≡
        **case** c **of**
          red → **true**,
          green → **false**
        **end**
  **end**





**type** Colour == red | green

is short for

**type** Colour
**value** red, green : Colour
**axiom**
  [Colour_disjoint] red ≠ green,

  [Colour_induction]
    ∀ p : Colour → **Bool** ·
      p(red) ∧ p(green) ⇒
        (∀ c : Colour · p(c))

## Wildcards

**type** Colour == red | green | _

is short for

**type** Colour
**value** red, green : Colour
**axiom**
[Colour_disjoint] red ≠ green

The induction axiom disappears. Further colours may be added during development.





## Variant definitions 2

```
TREE =
  class
    type
      Tree ==
        nil |
        node(left : Tree, val : Elem, right : Tree),
      Elem

    value
      traverse : Tree → Elem*
      traverse(t) ≡
        case t of
          nil → ⟨⟩,
          node(l, v, r) →
            traverse(l) ⌢ ⟨v⟩ ⌢ traverse(r)
        end
  end
```





```
type
  Tree ==
    nil |
    node(left : Tree, val : Elem, right : Tree)
```

is short for

```
type Tree
value
  nil : Tree,
  node : Tree × Elem × Tree → Tree,
  left : Tree ~→ Tree,
  val  : Tree ~→ Elem,
  right : Tree ~→ Tree
axiom
  [left_node]
    ∀ l, r : Tree, v : Elem ·
      left(node(l, v, r)) ≡ l,
  [val_node]
    ∀ l, r : Tree, v : Elem ·
      val(node(l, v, r)) ≡ v,
  [right_node]
    ∀ l, r : Tree, v : Elem ·
      right(node(l, v, r)) ≡ r,
```





```
  [Tree_disjoint]
    ∀ l, r : Tree, v : Elem ·
      nil ≠ node(l, v, r),
  [Tree_induction]
    ∀ p : Tree → Bool ·
      p(nil) ∧
      (∀ l, r : Tree, v : Elem ·
        p(l) ∧ p(r) ⇒ p(node(l, v, r))) ⇒
        (∀ t : Tree · p(t))
```

# Imperative Specification





## Imperative Specification Example

COUNTER =
  **class**
    **variable**
      counter : **Nat** := 0
    **value**
      increase : **Unit** → **write** counter **Nat**
      increase() ≡ counter := counter + 1 ; counter
  **end**





## Variable Definitions

**variable**
  variable_definition$_1$,
  ⋮
  variable_definition$_n$

single variable definition:

id : type_expr := value_expr

multiple variable definition:

id$_1$,...,id$_n$ : type_expr





## Functions with Variable Access

Function types revisited:

type_expr$_1$ $\xrightarrow{\sim}$ access_desc$_1$ ... access_desc$_n$ type_expr$_2$

access_desc$_i$:

- **read** id$_1$,...,id$_n$
- **write** id$_1$,...,id$_n$

Function definitions:

bodies must not statically access variables
which are not mentioned in the access descriptions

## Imperative Expressions

No syntactic distinction between

- statements and
- expressions

Imperative expressions:

- assignments (id := value_expr)
- sequencing ($unit$-value_expr$_1$ ; value_expr$_2$)
- repetitive expressions (while, until, for)
- if expressions
- ...

Imperative expressions may have effects

*effect* = state change

*state* = particular contents of variables





## Pure and Read-only

Properties of value expressions:

- *read* a variable
- *write* to a variable (e.g. x := e writes to x)
- *access* a variable: read or write to a variable
- *pure*: does not statically access any variable (e.g. 5)
- *read-only*: does not statically write to any variable (e.g. 5, x + 1)

Context conditions:

- must be pure

  {| binding : type_expr · $pure$-value_expr |}

- must be read-only

  {$readonly$-value_expr$_1$ .. $readonly$-value_expr$_2$}





## Evaluation Order

- left to right

The order has particular importance
when the constituent expressions have effects

Examples:

Given **variable** x : **Int**

$\langle$x := 1 ; x , x := 2 ; x$\rangle$ ≡ x := 2 ; $\langle$1,2$\rangle$
$\langle$x := 2 ; x , x := 1 ; x$\rangle$ ≡ x := 1 ; $\langle$2,1$\rangle$

x + (x := x + 1 ; x) ≡ x := x + 1 ; 2 ∗ x − 1
(x := x + 1 ; x ) + x ≡ x := x + 1 ; 2 ∗ x





## Equivalence versus Equality

= and ≡ differ in terms of

- undefinedness (**chaos**)
- non-determinism
- effects (variables and communication)

otherwise they are the same.

For example, we can say

factorial(3) = 6
or
factorial(3) ≡ 6

They are both true

## When equivalence and equality differ

Assume the variable x currently holds the value 0.

| Expression | Evaluation |
|---|---|
| $1 \parallel 2 = 1 \parallel 2$ | **true** $\parallel$ **false** |
| $1 \parallel 2 \equiv 1 \parallel 2$ | **true** |
| **while true do skip end** = **chaos** | **chaos** |
| **while true do skip end** $\equiv$ **chaos** | **true** |
| $((x := x + 1 ; 1) = (x := x + 1 ; x))$ | $x := 2$ ; **false** |
| $((x := x + 1 ; 1) \equiv (x := x + 1 ; x))$ | **true** |





- $\equiv$ and $=$ are the same if
  the arguments are convergent and pure.
- $\equiv$ is always defined.
- $\equiv$ compares effects as well as results;
  $=$ only compares results
- $\equiv$ has hypothetical evaluation;
  $=$ has left-to-right evaluation.
- $\equiv$ gives no effects;
  $=$ may give effects.





## Imperative Preconditions

Example:

```
DECREASE =
  extend COUNTER with
  class
    value
      decrease : Unit ~→ write counter Nat
      decrease() ≡ counter := counter − 1 ; counter
      pre counter > 0
  end
```

preconditions must be read-only.





## Some Examples

```
TEST_COUNTER =
  extend COUNTER with
  class
    value
      increase_and_test : Nat → write counter Bool
      increase_and_test(n) ≡ increase() ≤ n
  end
```

```
INCREASE_TWICE =
  extend COUNTER with
  class
    value
      increase_twice : Unit → write counter Nat
      increase_twice() ≡ increase() ; increase()
  end
```

is wrong

```
INCREASE_TWICE =
  extend COUNTER with
  class
    value
      increase_twice : Unit → write counter Nat
      increase_twice() ≡
        let dummy = increase() in increase() end
  end
```

is correct





```
COUNTER =
  class
    variable
      counter : Nat := 0
    value
      increase : Unit → write counter Unit
      increase() ≡ counter := counter + 1,

      return_counter : Unit → read counter Nat
      return_counter() ≡ counter
  end
```

```
INCREASE_TWICE =
  extend COUNTER with
  class
    value
      increase_twice : Unit → write counter Nat
      increase_twice() ≡
        increase() ; increase() ; return_counter()
  end
```





## Applicative to imperative transformation

- Remove definition of type of interest.
- Add variable(s) to model concrete type of interest.
- Remove type of interest from function signatures; fill holes with **Unit**.
- Give type "**Unit** → **write** ... **Unit**" to each constant "c" of type of interest, and replace "=" by "c() ≡".
- Insert write accesses to generators.
- Insert read accesses to observers.
- Remove formal parameters representing type of interest.
- Replace occurrences of type of interest parameters with references to variable(s).
- For generators, insert assignments.

This is for "leaf" modules in a hierarchy. We will see later an example of how to transform non-leaf modules.





```
I_DATABASE =
  class
    type
      Key, Data
    variable
      database : Key ~m~> Data
    value
      empty : Unit → write database Unit
      empty() ≡ database := [ ],

      insert : Key × Data → write database Unit
      insert(k,d) ≡ database := database † [ k ↦ d ],

      remove : Key → write database Unit
      remove(k) ≡ database := database \ {k},

      defined : Key → read database Bool
      defined(k) ≡ k ∈ dom database,

      lookup : Key ~~> read database Data
      lookup(k) ≡ database(k)
      pre defined(k)
   end
```

## If Expressions Revisited

Standard form:

**if** value_expr$_1$ **then** value_expr$_2$ **else** value_expr$_3$ **end**

Derived form:

**if** value_expr$_1$ **then** value_expr$_2$ **end**

short for:

**if** value_expr$_1$ **then** value_expr$_2$ **else skip end**

where

**skip** ≡ ()

Example:

```
variable counter : Nat
value
  decrease : Unit → write counter Unit
  decrease() ≡
    if counter > 0 then counter := counter − 1 end
```





## While Expressions

General form:

**while** *logical*-value_expr$_1$ **do** *unit*-value_expr$_2$ **end**

has type **Unit**





1 + 1/2 + ... + 1/n

```
FRACTION_SUM =
  class
    variable
      counter : Nat,
      result : Real
    value
      fraction_sum : Nat ~→ write counter, result Unit
      fraction_sum(n) ≡
        counter := n ;
        result := 0.0 ;
        while counter > 0 do
          result := result + 1.0/(real counter) ;
          counter := counter − 1
        end
      pre n > 0
  end
```





## Until Expressions

General form:

**do** *unit*-value_expr$_1$ **until** *logical*-value_expr$_2$ **end**

has type **Unit**

```
FRACTION_SUM =
  class
    variable
      counter : Nat,
      result : Real
    value
      fraction_sum : Nat ~→ write counter, result Unit
      fraction_sum(n) ≡
        counter := n ;
        result := 0.0 ;
        do
          result := result + 1.0/(real counter) ;
          counter := counter − 1
        until counter = 0 end
      pre n > 0
  end
```





## For Expressions

General form:

**for**
 binding
**in**
  *readonly_list*−value_expr$_1$ · *readonly_logical*−value_expr$_p$
**do**
  *unit*-value_expr$_2$
**end**

has type **Unit**





```
FRACTION_SUM =
  class
    variable
      result : Real
    value
      fraction_sum : Nat ~→ write result Unit
      fraction_sum(n) ≡
        result := 0.0 ;
        for i in ⟨1 .. n⟩ do
          result := result + 1.0/(real i)
        end
      pre n > 0
  end
```





## Local Expressions

General form:

**local**
  declaration$_1$ ... declaration$_n$
**in** value_expr **end**

```
FRACTION_SUM =
  class
    value
      fraction_sum : Nat ~→ Real
      fraction_sum(n) ≡
        local
          variable
            counter : Nat := n,
            result : Real := 0.0
          value
            calc_fraction :
              Unit → write counter, result Unit
            calc_fraction() ≡
              result := result + 1.0/(real counter) ;
              counter := counter − 1
        in
          while counter > 0 do calc_fraction() end ; result
        end
      pre n > 0
  end
```





## Local versus Let

```
local
  declaration1 . . . declarationn
in value_expr end

let
  let_def1, . . . ,let_defn
in value_expr end
```

In local expressions:

- local declarations of:
  all kinds (not just of values)
- scope of declaration$_i$:
  all the declarations
  (not just the declaration$_j$, j > i)





## Pre-names

```
CHOOSE =
  class
    variable
      set : Int-set
    value
      choose : Unit ~→ write set Int
      choose() as i post i ∈ set` ∧ set = set`\{i}
      pre set ≠ {}
  end
```

Avoiding pre-names:

```
value
  choose : Unit ~→ write set Int
axiom
  let s = set in
    choose() as i post i ∈ s ∧ set = s\{i}
    pre set ≠ {}
  end
```





```
INSERT_SORTED =
  class
    variable
      list : Int* := ⟨⟩
    value
      insert : Int → write list Unit
      insert(i)
        post is_permutation(list, ⟨i⟩ ⌢ list`) ∧ is_sorted(list)

      /∗ auxiliary functions ∗/
      is_sorted : Int* → Bool,
      is_sorted(l) ≡
        (∀ idx1,idx2 : Nat ·
          ({idx1,idx2} ⊆ inds l ∧ idx1 < idx2) ⇒
            l(idx1) ≤ l(idx2)),

      is_permutation : Int* × Int* → Bool
      is_permutation(l1,l2) ≡
        (∀ i : Int · count(i, l1) = count(i, l2)),

      count : Int × Int* → Nat
      count(i, l) ≡
        card {idx | idx : Nat ·
                idx ∈ inds l1 ∧ l1(idx) = i}

  end
```

# Concurrency





## Agenda

- channel definitions (**channel** c : T)
- (process) expressions:
  - concurrency (e1 ‖ e2)
  - communication (c?, c!e)
  - choice (e1 [] e2, e1 ⌈⌉ e2)
  - **stop** (deadlock)
- functions with channel access
  (value f : **Unit** $\xrightarrow{\sim}$ **in** c **out** c **Unit**)
  - applicative versus imperative





## Composition of Expressions

Composition:

- sequential:

  $\text{value_expr}_1$ ; $\text{value_expr}_2$

- concurrent:

  $\text{value_expr}_1$ ‖ $\text{value_expr}_2$

  1. has type **Unit**
  2. $\text{value_expr}_1$ and $\text{value_expr}_2$ must have type **Unit**
  3. $\text{value_expr}_1$ and $\text{value_expr}_2$ recommended to be assignment-disjoint





## Concurrency

Concurrency is necessary in particular for describing distributed systems.

Concurrent systems in general may communicate through

- shared variables
- message passing

RSL uses message passing.

## Channel Definitions

**channel**
  channel_definition$_1$,
  ⋮
  channel_definition$_n$

single channel definition:

  id : type_expr

multiple channel definition:

  id$_1$,...,id$_n$ : type_expr





## Communication Expressions

**channel** id : type_expr

Communication expressions:

- input expressions: id ?
- output expressions: id ! value_expr





## Example

**channel** c : **Int**
**variable** x : **Int**

... x := c? ‖ c!5 ...

communication may

1. be synchronized: x := 5
2. be interleaved, if other behaviours are possible

**channel** c : **Int**
**variable** x : **Int**

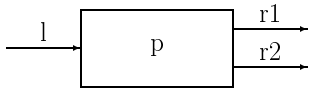
... (x := c? ‖ c!5) ‖ c!7 ...

may give: x := 7; c!5 or ...





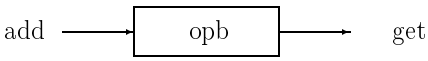
## Example




**channel**
  l, r1, r2: **Int**

**value**
  p : **Unit** → **in** l **out** r1, r2 **Unit**
  p() ≡
    **let** e = l? **in** (r1!e ‖ r2!e) **end**; p()

## Example




```
ONE_PLACE_BUFFER =
  class
    type Elem
    channel add, get : Elem
    value
      opb : Unit → in add out get Unit
      opb() ≡ let v = add? in get!v end ; opb()
  end
```





## Functions with Channel Access

Function types revisited:

$$\text{type_expr}_1 \xrightarrow{\sim} \text{access_desc}_1 \ldots \text{access_desc}_n\ \text{type_expr}_2$$

$\text{access_desc}_i$:

- **read** $\text{id}_1,\ldots,\text{id}_n$
- **write** $\text{id}_1,\ldots,\text{id}_n$
- **in** $\text{id}_1,\ldots,\text{id}_n$
- **out** $\text{id}_1,\ldots,\text{id}_n$

Function definitions:

bodies must not statically access variables or channels which are not mentioned in the access descriptions





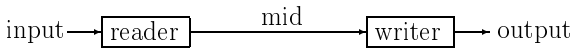
## Pure and Read-only Revisited

Properties of value expressions:

- *read* a variable
- *write* to a variable (e.g. x := e writes to x)
- *access* a variable: read or write to a variable
- *input* from a channel (e.g c? )
- *output* to a channel (e.g. c!e)
- *access* a channel: input from or output to a channel
- *pure*: does not statically access any variable or channel
- *read-only*: does not statically write to any variable and does not statically access any channel








```
READER_WRITER =
  class
    type Elem
    channel input, output, mid : Elem
    value
      reader : Unit → in input out mid Unit
      reader() ≡
        let v = input? in mid ! v end ; reader(),
      writer : Unit → in mid out output Unit
      writer() ≡
        let v = mid? in output ! v end ; writer()
  end

SYSTEM = extend READER_WRITER with
  class
    value
      system : Unit →
                 in input, mid out output, mid Unit
      system() ≡ reader() ‖ writer()
  end
```

```
system() ≡
  let v = input? in mid ! v end ; reader()
  ‖
  let v = mid? in output ! v end ; writer()
```

We should make the channel *mid* unavailable to any other processes.





## Restricting channel access

```
local
  channel c : Int
in x := c? ‖ c!5 end
≡
x := 5
```





## Deadlock

```
local
  channel c : Int
in  c!2 ‖ c!5 end
≡
stop
```

Note the difference between **stop** and **skip**.

**skip** *terminates*, which means the next expression in sequence may execute.

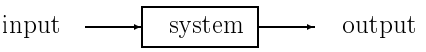
**stop** does not terminate.

So

```
skip ; e ≡ e
stop ; e ≡ stop
```








```
SYSTEM =
  class
    type Elem
    channel input, output : Elem
    value
      system : Unit → in input out output Unit
      system() ≡
        local
          channel mid : Elem
          value
            reader : Unit → in input out mid Unit
            reader() ≡
              let v = input? in mid ! v end ; reader(),
            writer : Unit → in mid out output Unit
            writer() ≡
              let v = mid? in output ! v end ; writer()
        in reader() ‖ writer() end
  end
```

## External choice

General form:

value_expr$_1$ [] value_expr$_2$

Example:

- The value expression

  v:=c1? [] c2!e

  will:

  – input from $c_1$ if a value expression is willing to output to $c_1$ but no value expression is willing to input from $c_2$;

  – output to $c_2$ if a value expression is willing to input from $c_2$ but no value expression is willing to output to $c_1$;

  – either input from $c_1$ or output to $c_2$ if a value expression is willing to output to $c_1$ and a value expression is willing to input from $c_2$;

  – deadlock if no value expression is ever willing to output to $c_1$ and no value expression is ever willing to input from $c_2$.





## Internal choice

General form:

value_expr$_1$ |~| value_expr$_2$

Example:

- The value expression

  v:=c1? |~| c2!e

  will:

  – either deadlock or input from $c_1$ if a value expression is willing to output to $c_1$ but no value expression is willing to input from $c_2$;

  – either deadlock or output to $c_2$ if a value expression is willing to input from $c_2$ but no value expression is willing to output to $c_1$;

  – either input from $c_1$ or output to $c_2$ if a value expression is willing to output to $c_1$ and a value expression is willing to input from $c_2$;

  – deadlock if no value expression is ever willing to output to $c_1$ and no value expression is ever willing to input from $c_2$.





```
MANY_PLACE_BUFFER =
  class
    type
      Elem,
      Buffer = Elem*
    channel
      empty : Unit,
      add, get : Elem
    value
      mpb : Buffer → in empty, add out get Unit
      mpb(b) ≡
        empty? ; mpb(⟨⟩)
        []
        let v = add? in mpb(b ⌢ ⟨v⟩) end
        []
        if b ≠ ⟨⟩
          then get ! hd b ; mpb(tl b)
          else stop end
  end
```



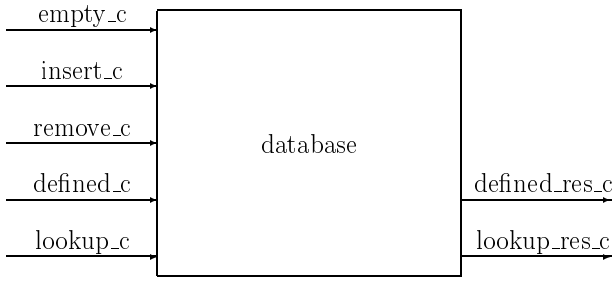


```
  mpb(⟨⟩)
≡
  empty? ; mpb(⟨⟩)
  []
  let v = add? in mpb(⟨⟩ ⌢ ⟨v⟩) end
  []
  if ⟨⟩ ≠ ⟨⟩ then get ! hd ⟨⟩ ; mpb(tl ⟨⟩) else stop end
≡
  empty? ; mpb(⟨⟩)
  []
  let v = add? in mpb(⟨v⟩) end
  []
  stop
≡
  empty? ; mpb(⟨⟩)
  []
  let v = add? in mpb(⟨v⟩) end
```

```
MANY_PLACE_BUFFER =
  class
    type
      Elem
    channel
      empty : Unit,
      add, get : Elem
    value
      mpb : Unit → in empty, add out get Unit
      mpb() ≡
        local
          type Buffer = Elem*
          variable buffer : Buffer := ⟨⟩
        in
          while true do
            empty? ; buffer := ⟨⟩
            []
            let v = add? in buffer := buffer ⌢ ⟨v⟩ end
            []
            if buffer ≠ ⟨⟩
            then get ! hd buffer ; buffer := tl buffer
            else stop
            end
          end
        end
  end
```












## Imperative to concurrent transformation

- Insert an object instantiating the imperative sequential module, and hide it.
- Define channels for each observer and generator; at least one channel for each. Hide them.
- Define a "server" process:
  - type "**Unit → in** ... **out** ... **write** I.**any Unit**"
  - body is a **while true do** loop
  - loop body is an external choice between clauses, one clause for each observer and each generator
  - each clause inputs parameters (if any); calls corresponding function I.f; outputs result (if any). Must do at least one communication.

  Hide it.
- Define an "init" process with the same type as the server that initialises the imperative object and calls the server.
- Define "interface functions" mirroring clauses in server. These *have no accesses to the imperative object.*

This is for "leaf" modules in a hierarchy. We will see later an example of how to transform non-leaf modules.





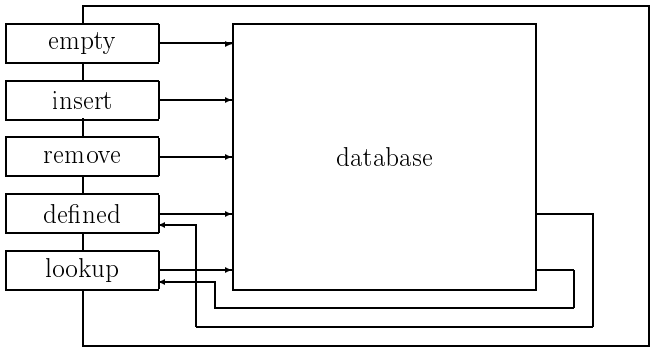
```
C_DATABASE =
  hide I, database in
  class
    object I : I_DATABASE
    type
      Key = I.Key,
      Data = I.Data,
      Result == not_found | res(Data)
    channel
      empty_c : Unit,
      insert_c : Key × Data,
      remove_c, defined_c, lookup_c : Key,
      defined_res_c : Bool,
      lookup_res_c : Result
    value
      init : Unit →
        in empty_c, insert_c, remove_c, defined_c, lookup_c
        out defined_res_c, lookup_res_c write I.any Unit
      init() ≡ I.empty() ; database(),

      database : Unit →
        in empty_c, insert_c, remove_c, defined_c, lookup_c
        out defined_res_c, lookup_res_c write I.any Unit
```

```
  database() ≡
    while true do
      empty_c? ; I.empty()
      □
      let (k,d) = insert_c? in I.insert(k,d) end
      □
      let k = remove_c? in I.remove(k) end
      □
      let k = defined_c? in
        defined_res_c ! I.defined(k)
      end
      □
      let k = lookup_c? in
        if I.defined(k)
        then lookup_res_c ! res(I.lookup(k))
        else lookup_res_c ! not_found
        end
      end
    end
end
```





## Encapsulation with Interface Functions








```
INTERFACED_DATABASE =
  hide empty_c, insert_c, remove_c, defined_c, lookup_c,
       defined_res_c, lookup_res_c in
  extend C_DATABASE with
  class
    value
      empty : Unit → out any Unit
      empty() ≡ empty_c ! (),

      insert : Key × Data → out any Unit
      insert(k,d) ≡ insert_c ! (k,d),

      remove : Key → out any Unit
      remove(k) ≡ remove_c ! k,

      defined : Key → in any out any Bool
      defined(k) ≡ defined_c ! k ; defined_res_c?,

      lookup : Key → in any out any Result
      lookup(k) ≡ lookup_c ! k ; lookup_res_c?
  end
```





## Modularity

## Specifications

An RSL specification consists of

- module definitions

A module contains definitions of

- types
- values
- variables
- channels
- modules
- axioms





## Modularity

Modules are the building blocks.

Purposes:

- Readability
- Separate development
- Reuse





## Schemes and Objects

Modules are either schemes or objects.

A scheme denotes a class of models

**scheme** id = class_expr

An object denotes a single model

**object** id : class_expr





## Class Expressions

- basic
- extending
- renaming
- hiding
- instantiation

## Extension

General form:

**extend** class_expr$_1$ **with** class_expr$_2$

class_expr$_1$ and class_expr$_2$ must be compatible

Context dependent expansion:

**extend**
  **class** decl_string$_1$ **end**
**with**
  **class** decl_string$_2$ **end**

expands to:

**class**
  decl_string$_1$
  decl_string$_2$
**end**





## Renaming

General form:

**use**
  id$_{\text{new}_1}$ **for** id$_{\text{old}_1}$, ... ,id$_{\text{new}_n}$ **for** id$_{\text{old}_n}$
**in** class_expr

For example

**scheme** BUFFER =
  **use**
    add **for** enq, get **for** deq, Buffer **for** Queue
  **in** QUEUE





## Hiding

General form:

**hide** id$_1$,...,id$_n$ **in** class_expr

Hidden entities

1. are not visible outside
2. need not be implemented

Typically use:

1. prevention of unintended access to variables and/or channels
2. hiding of auxiliary functions





## Objects

**scheme** BUFFER =
  **class**
    **variable** buff : **Int**$^*$
    **value**
      is_empty : **Unit** → **read** buff **Bool**
      ...
  **end**

**object**
  B1 : BUFFER,
  B2 : BUFFER,

**scheme** SYS =
  **class**
    **value**
      one_is_empty :
        **Unit** → **read** B1.buff B2.buff **Bool**
      one_is_empty() ≡
        B1.is_empty() ∨ B2.is_empty()
  **end**

B1.buff and B2.buff are distinct

## Module Nesting

```
scheme
  SYS =
    class
      object
        B1 : BUFFER,
        B2 : BUFFER
      value
        one_is_empty :
          Unit → read B1.buff B2.buff Bool
        one_is_empty() ≡
          B1.is_empty() ∨ B2.is_empty()
    end
```





## Building hierarchies

Suppose we have a system that needs a database component.

There are several ways we can construct the specification:

- merging the system and database definitions in one class
- extending the database class with the system class
- making a hierarchy with a database object





## Merging the definitions in one class

```
scheme SYSTEM =
  class
    /* database */
    ⋮
    /* system */
    ⋮
  end
```

- Hard to read
- Database cannot be reused
- Hard to make database private to system
- Problem of name clashes between two parts





## Extending the database

```
scheme DATABASE = ...

scheme SYSTEM =
  extend DATABASE with ...
```

- Easier to read
- Database can be reused
- Hard to make database private to system
- Problem of name clashes between two parts

## Making a hierarchy with a database object

**scheme** DATABASE = ...

**scheme** SYSTEM =
  **class**
    **object** DB : DATABASE
    ⋮
  **end**

- Easier to read
- Database can be reused
- Easy to make database private to system

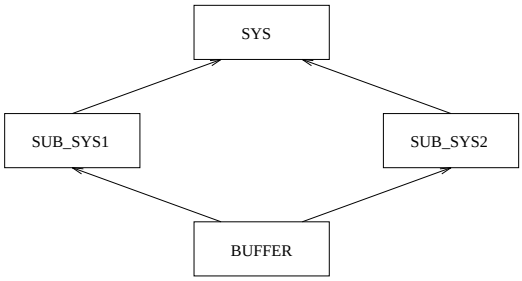
  **scheme** SYSTEM =
    **hide** DB **in**
    **class**
      **object** DB : DATABASE
      ⋮
    **end**

- No problem of name clashes between two parts





## Sharing




**scheme** BUFFER = ...

**scheme** SUB_SYS1 =
  **class object** B : BUFFER ... **end**
**scheme** SUB_SYS2 =
  **class object** B : BUFFER ... **end**

**scheme** SYS =
  **class**
    **object**
      O1 : SUB_SYS1,
      O2 : SUB_SYS2
    ⋮
  **end**

We get two buffer variables (O1.B.buff and O2.B.buff)





## Sharing using Global Objects

**object**
  B : BUFFER

**scheme**
  SUB_SYS1 = **class** ... B.buff ... **end**,
  SUB_SYS2 = **class** ... B.buff ... **end**,

  SYS =
    **class**
      **object**
        O1 : SUB_SYS1,
        O2 : SUB_SYS2
    **end**

We get only one buffer: B.buff





## Parameterization - Example

**scheme** BUFFER =
  **class**
    **type** Elem
    **variable** buff : Elem*
    **value**
      empty : **Unit** → **write** buff **Unit**
      empty() ≡ buff := ⟨⟩,

      add : Elem → **write** buff **Unit**
      add(e) ≡ buff := buff ⌢ ⟨e⟩
  **end**

is better expressed using parameterization

**scheme** ELEM = **class type** Elem **end**

**scheme** BUFFER(E : ELEM) =
  **class**
    **variable** buff : E.Elem*
    **value**
      empty : **Unit** → **write** buff **Unit**
      empty() ≡ buff := ⟨⟩,

      add : E.Elem → **write** buff **Unit**
      add(e) ≡ buff := buff ⌢ ⟨e⟩
  **end**

## Instantiation - Example

**object**
  INTEGER :
    **class**
      **type** Elem = **Int**
    **end**,

  INTEGER_BUFFER :  BUFFER(INTEGER)

If we expand INTEGER_BUFFER:

  INTEGER_BUFFER :
    **class**
      **variable** buff : INTEGER.Elem*
      **value**
        empty : **Unit** → **write** buff **Unit**
        empty() ≡ buff := ⟨⟩,

        add : INTEGER.Elem → **write** buff **Unit**
        add(e) ≡ buff :=  buff ⌢ ⟨e⟩
    **end**





## Actual versus Formal Parameters

  **scheme** S(X : FC)
  **object** A : AC,

... S(A) ...

Context condition: AC must statically implement FC





## Sharing by Parameterization

**scheme**
  SUB_SYS1(B : BUFFER) =
    **class** ... B.buff ... **end**,

  SUB_SYS2(B : BUFFER) =
    **class** ... B.buff ... **end**,

  SYS =
    **class**
      **object**
        B : BUFFER,
        O1 : SUB_SYS1(B),
        O2 : SUB_SYS2(B)
    **end**





## Summary

Modules:

- schemes
- objects

Class expressions:

- basic
- extending
- hiding
- renaming
- instantiation

# RAISE Method





## Agenda

1. Why formal methods?
2. Characteristics of formal methods
3. RAISE method
4. Implementation relation: requirements and definition
5. Example





## Why formal methods?

To produce software that is

- more likely to be correct
- more reliable
- better documented
- more easily maintainable





## What is formality?

- a language — symbols and grammar rules for constructing terms
- (usually) rules for deciding if terms are well formed (e.g. scope, typing rules)
- a semantics — a description of what terms mean
- a logic — a set of rules for determining if predicates about terms are true

Programming languages are not formal according to this definition because they lack a logic.

## Characteristics of formal methods

- Precise notation
- Abstraction (*what* rather than *how*)
- Stepwise development (gradual commitment)
- Proof opportunities and justifications
- Structuring based on compositionality
- Guidelines for quality assurance





## Rigorous methods

Choice of level of formality. E.g.

1. No proof opportunities generated or checked
2. Proof opportunities generated and inspected but not proved
3. Proof opportunities generated and proved with some informal steps — "it follows immediately that ..."
4. Proof opportunities generated and proved formally

All formal methods are in fact rigorous. But only a method with a formal basis can be rigorous, because it must always be possible to say "I am not sure if it does follow. Please prove it."

Current state of the art is the first three levels.





## Why formality?

When we try to be formal, what are our objectives?

- To provide unambiguous specification techniques
- To provide a theory to support reliable reasoning
- To provide descriptions in precise and machine independent terms





## Examples of formal methods

- "Model based" approaches like Z and VDM
- "Algebraic" like Larch and OBJ
- "Process algebras" like CSP, CCS and Lotos
- RAISE
- "Modal" logics, especially temporal logics for concurrency

RAISE contains most of the features of the first three

## Validation

### Are we building the "right" system?

- the specification is *validated* against the requirements
- a specification may be viewed as a *theory* about the requirements. Viewed in this way we can:
  - look at the requirements, identify a "requirements issue", and see if the specification "predicted" this requirement
  - look at the specification, deduce a property from the specification, and check the requirements to see if it is a valid prediction.





## Verification

### Are we building the system "right"?

The *implementation relation* captures the notion of correctness of a development step.

An assertion of implementation may be *statically* checked by tools. This shows that the *signature* has been maintained, e.g.

- no entities (types, values etc.) have been omitted
- the types of values, variables etc. have not been changed

If statically correct, the assertion may be *dynamically* checked by proof (or "justification"). This shows that *properties* have been maintained, e.g.

- subtypes have not changed
- axioms are true
- postconditions are established by concrete definitions

An implementation may have more entities and/or properties; it may not have fewer or weaker or different ones.





## The RAISE method

The method is based on 2 key notions:

**Separate development**

- Divide systems into subsystems
- Take the initial specification of the subsystem as a "contract" between its developers and the system developers
- Develop parts separately. Implementation rules mean that as long as contracts are not changed integration will preserve the original properties.

**Stepwise development** Repeatedly:

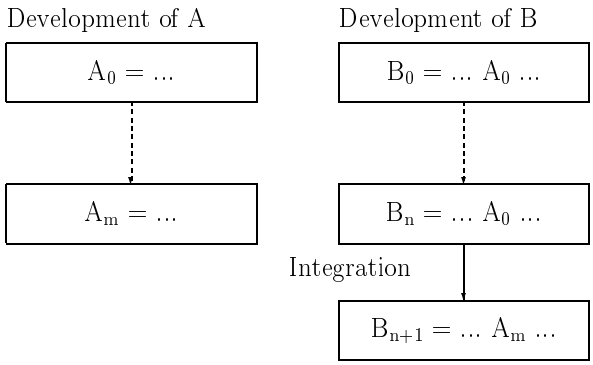
- Specify a new, more detailed version
- Assert the relation (if possible implementation) between this and the previous version, and justify that it holds

This follows the "invent and verify" paradigm.

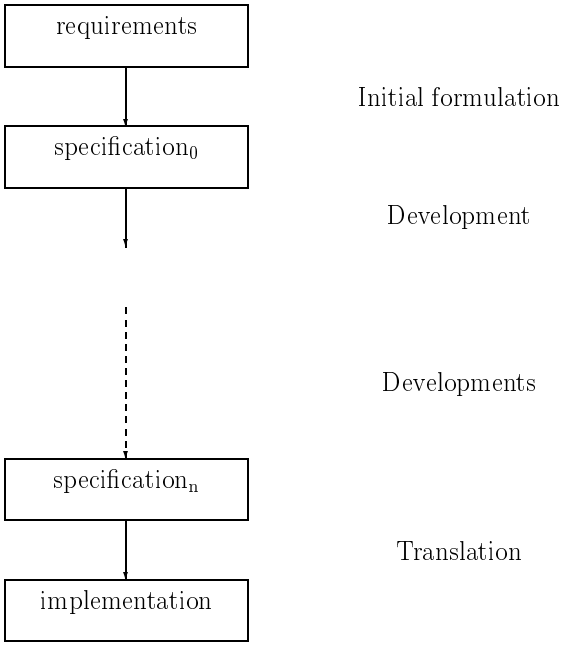




## Separate Development



## Stepwise development








## Implementation relation: requirements

- Property preservation
- Substitutivity
- Transitivity

These are needed for separate and stepwise development.





## Implementation relation

- new signature includes the old one (statically decidable)
- old properties preserved by the new one ($\Rightarrow$ implementation conditions)





## Underspecification

**value** x : **Int**
**axiom** x $>$ 0

x is underspecified, may be implemented by:

**value** x : **Int** = 1

or

**value** x : **Int** = 2

or ...

## Example 1

**scheme** S0 =
  **class**
    **value** x : **Int**
    **axiom** x ≥ 0
  **end**

**scheme** S1 =
  **class**
    **value**
      x : **Int** = 2
  **end**

**scheme** S2 =
  **class**
    **value**
      x : **Int** = 2
      y : **Int** = 0
  **end**

Does S1 or S2 implement S0?

Does S2 implement S1?





## Example 2

**scheme** S0 =
  **hide** z **in class**
    **value** x, y, z : **Int**
    **axiom** x > z ∧ z > y
  **end**

**scheme** S1 =
  **class**
    **value**
      x : **Int** = 1
      y : **Int** = 0
  **end**

**scheme** S2 =
  **class**
    **value**
      x : **Int** = 2
      y : **Int** = 0
  **end**

Does S1 or S2 implement S0?





## Showing implementation with hidden entities

1. Make an extension defining the hidden entities.
2. Show the extension is conservative.
3. Show the extension implements the old.

For example:

1. Define

   **scheme** S2′ =
     **extend** S2 **with**
     **hide** z **in class value** z : **Int** = 1 **end**

2. Extension is conservative since definition of $z$ cannot affect $x$ or $y$.
3. We can prove S2′ implements S0.





## Example RSL specification

**scheme** REGISTRATION1 =
  **class**
    **type**
      Register = Name-**set**,
      Name = **Text**

    **value**
      enrol : Name × Register → Register
      enrol(n,r) ≡ r ∪ {n},

      leave : Name × Register → Register
      leave(n,r) ≡ r \ {n},

      registered : Name × Register → **Bool**
      registered(n,r) ≡ n ∈ r
  **end**

## An abstract version of REGISTRATION

**scheme** REGISTRATION0 =
**class**
**type** Register, Name
**value**
/∗ generators ∗/
empty : Register,
enrol : Name × Register → Register,
leave : Name × Register → Register
/∗ observers ∗/
registered : Name × Register → **Bool**





## REGISTRATION0 continued

**axiom**
[ registered_empty ]
∀ n : Name ·
registered(n, empty) ≡ **false**,

[ registered_enrol ]
∀ n, n′ : Name, r : Register ·
registered(n′, enrol(n, r)) ≡
n′ = n ∨ registered(n′ , r),

[ registered_leave ]
∀ n, n′ : Name, r : Register ·
registered(n′, leave(n, r)) ≡
n′ ≠ n ∧ registered(n′ , r)
**end**

Each axiom relates an observer to a generator.





## Showing implementation for REGISTRATION Assert the relation:

"REGISTRATION1 ⪯ REGISTRATION0"
i.e.
"REGISTRATION1 implements REGISTRATION0"

### Static check

Signature of REGISTRATION0 included in REGISTRATION1 Checked by tool. False!

### Dynamic check

Properties of REGISTRATION0 hold in REGISTRATION1. Proof opportunities. E.g.

⌞ **in** REGISTRATION1 ⊢
∀ n, n′ : Name, r : Register ·
registered(n′, enrol(n, r)) ≡
n′ = n ∨ registered(n′ , r) ⌟

Reduces to

⌞ n′ ∈ (r ∪ {n}) ≡ n′ = n ∨ n′ ∈ r ⌟

This can be proved.

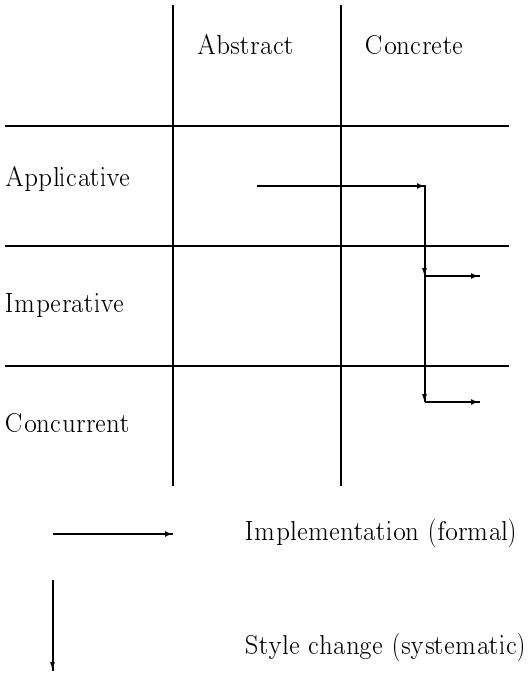




## Design

- removing underspecification
  - abstract types to concrete types
  - more explicit value definitions
- changing style
  - applicative/imperative
  - sequential/concurrent
- providing more efficient algorithms

## Typical Development








## A concrete, imperative version

```
scheme I_REGISTRATION1 =
  hide register, Register in class
    type
      Register = Name-set,
      Name = Text

    variable register : Register := {}

    value
      empty : Unit → write any Unit
      empty() ≡ register := {},

      enrol : Name → write any Unit
      enrol(n) ≡  register := register ∪ {n},

      leave : Name  → write any Unit
      leave(n) ≡  register := register \ {n},

      registered : Name  → read any Bool
      registered(n) ≡  n ∈ register
  end
```





## A concrete, concurrent version

```
scheme C_REGISTRATION1 =
  hide main, CH, I in class
    type
      Name = Text
    object
      CH :
        class
          channel
            empty : Unit,
            enrol, leave, registered : Name,
            registered_res : Bool
        end,
        I : I_REGISTRATION1
```

We first define some channels and an instantiation of the sequential imperative version.

The concurrent version will act as a communication shell around the sequential version.





## Concurrent version continued

```
value
  /∗ initial ∗/
  init : Unit → in any out any write any Unit
  init() ≡ I.empty() ; main(),

  /∗ main ∗/
  main : Unit → in any out any write any Unit
  main() ≡
    while true do
      CH.empty? ; I.empty()
      ▯
      let n = CH.enrol? in I.enrol(n) end
      ▯
      let n = CH.leave? in I.leave(n) end
      ▯
      let n = CH.registered? in
        CH.registered_res ! I.registered(n) end
  end
```

*init* is used to start the *main* "server" process.

## Concurrent version continued

**value**
  empty : **Unit → in any out any Unit**
  empty() ≡ CH.empty!(),

  enrol : Name → **in any out any Unit**
  enrol(n) ≡ CH.enrol!n,

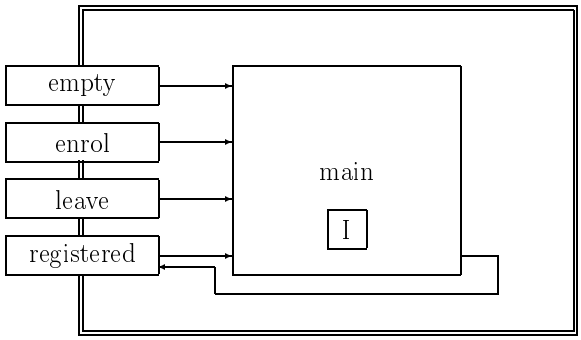
  leave : Name → **in any out any Unit**
  leave(n) ≡ CH.leave!n,

  registered : Name → **in any out any Bool**
  registered(n) ≡ CH.register!n ; CH.register_res?
**end**

Lastly add the 'interface' functions which provide the user interface to the registration system.












## Translation

- manual translation
- automatic translation (to Ada and C++)

of low-level RSL (e.g. concrete types and explicit value definitions)





## RAISE Method Summary

- separate development
- stepwise development
- rigorous
- invent and verify
- implementation relation

# Harbour example





## Requirements

Ships arriving at a harbour have to be allocated berths in the harbour which are vacant and which they will fit, or wait in a "pool" until a suitable berth is available.

Develop a system providing the following functions to allow the harbour master to control the movement of ships in and out of the harbour:

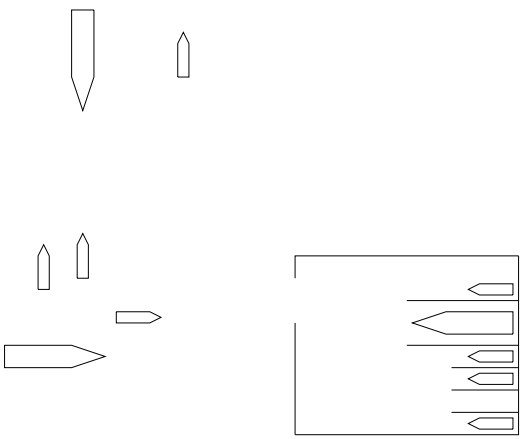
**arrive:** to register the arrival of a ship

**dock:** to register a ship docking in a berth

**leave:** to register a ship leaving a berth



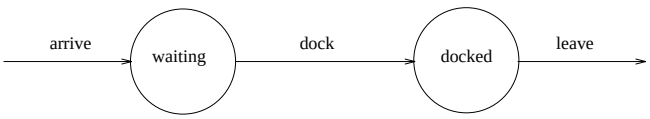




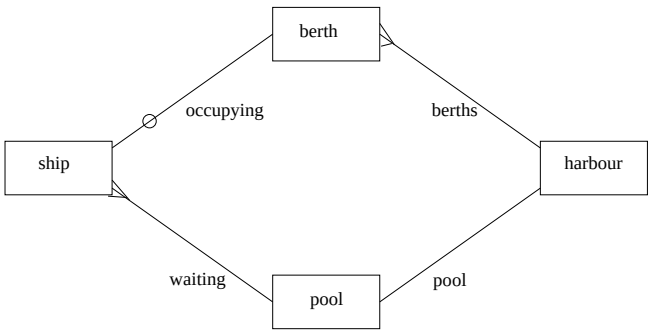




## State transitions for ships



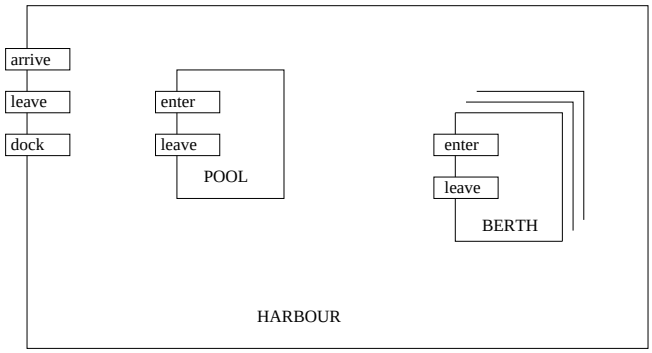
## Entity relationship diagram








## Harbour objects








## Possible attributes

- Harbour
  - Pool (S)
  - (Set of) berths (S)
- Pool
  - (Set of) ships (D)
- Berth
  - Occupancy (D)
  - Size (S)
- Ship
  - Location (D)
  - Name (S)
  - Size (S)

"S" indicates a static attribute
"D" indicates a dynamic (state-dependent) attribute





## Design decisions

- Don't know components of "size" — length, width, depth/draught etc. So define

  **value**
  fits : Ship × Berth → **Bool**

  and leave underspecified.
- Name of ship unnecessary
- Location of ship can be calculated (to avoid duplication)

## TYPES module

**scheme** TYPES =
  **class**
    **type**
      Ship, Berth,
      Occupancy == vacant | occupied_by(occupant : Ship)
    **value**
      fits : Ship × Berth → **Bool**
  **end**

We then make a global object from TYPES:

**object** T : TYPES





## Consistency

1. a ship can't be in two places at once
2. at most one ship can be in any one berth
3. a ship can only be in a berth it fits

Two possibilities:

- build into model
- express as a predicate

2nd consistency condition in *Occupancy*; for 1st and 3rd we will use a predicate.





## Abstract applicative specification

**scheme** A_HARBOUR0 =
**hide** consistent **in**
**class**
  **type** Harbour
  **value**
    /∗ generators ∗/
    arrives : T.Ship × Harbour $\xrightarrow{\sim}$ Harbour,
    docks : T.Ship × T.Berth × Harbour $\xrightarrow{\sim}$ Harbour,
    leaves : T.Ship × T.Berth × Harbour $\xrightarrow{\sim}$ Harbour,

    /∗ observers ∗/
    waiting : T.Ship × Harbour → **Bool**,
    occupancy : T.Berth × Harbour → T.Occupancy,





    /∗ derived ∗/
    consistent : Harbour → **Bool**
    consistent(h) ≡
      (∀ s : T.Ship ·
        ∼ (waiting(s, h) ∧ is_docked(s, h)) ∧
        (∀ b1, b2 : T.Berth ·
          occupancy(b1, h) = T.occupied_by(s) ∧
          occupancy(b2, h) = T.occupied_by(s) ⇒
            b1 = b2) ∧
        (∀ b : T.Berth ·
          occupancy(b, h) = T.occupied_by(s) ⇒
            T.fits(s, b))),

```
is_docked : T.Ship × Harbour → Bool
is_docked(s, h) ≡
  (∃ b : T.Berth ·
    occupancy(b, h) = T.occupied_by(s)),

/* guards */
can_arrive : T.Ship × Harbour → Bool
can_arrive(s, h) ≡
  ∼waiting(s, h) ∧ ∼is_docked(s, h),

can_dock : T.Ship × T.Berth × Harbour → Bool
can_dock(s, b, h) ≡
  waiting(s, h) ∧ ∼ is_docked(s, h) ∧
  occupancy(b, h) = T.vacant ∧ T.fits(s, b),

can_leave : T.Ship × T.Berth × Harbour → Bool
can_leave(s, b, h) ≡
  occupancy(b, h) = T.occupied_by(s)
```





```
axiom
  [waiting_arrives]
    ∀ h : Harbour, s1, s2 : T.Ship ·
      waiting(s2, arrives(s1, h)) ≡
        s1 = s2 ∨ waiting(s2, h)
      pre can_arrive(s1, h),

  [waiting_docks]
    ∀ h : Harbour, s1, s2 : T.Ship, b : T.Berth ·
      waiting(s2, docks(s1, b, h)) ≡
        s1 ≠ s2 ∧ waiting(s2, h)
      pre can_dock(s1, b, h),

  [waiting_leaves]
    ∀ h : Harbour, s1, s2 : T.Ship, b : T.Berth ·
      waiting(s2, leaves(s1, b, h)) ≡
        waiting(s2, h)
      pre can_leave(s1, b, h),
```





```
  [occupancy_arrives]
    ∀ h : Harbour, s : T.Ship, b : T.Berth ·
      occupancy(b, arrives(s, h)) ≡
        occupancy(b, h)
      pre can_arrive(s, h),

  [occupancy_docks]
    ∀ h : Harbour, s : T.Ship, b1, b2 : T.Berth ·
      occupancy(b2, docks(s, b1, h)) ≡
        if b1 = b2 then T.occupied_by(s)
        else occupancy(b2, h) end
      pre can_dock(s, b1, h),

  [occupancy_leaves]
    ∀ h : Harbour, s : T.Ship, b1, b2 : T.Berth ·
      occupancy(b2, leaves(s, b1, h)) ≡
        if b1 = b2 then T.vacant
        else occupancy(b2, h) end
      pre can_leave(s, b1, h),
```





```
  [arrives_consistent]
    ∀ h : Harbour, s : T.Ship ·
      arrives(s, h) as h′ post consistent(h′)
        pre consistent(h) ∧ can_arrive(s, h),

  [docks_consistent]
    ∀ h : Harbour, s : T.Ship, b : T.Berth ·
      docks(s, b, h) as h′ post consistent(h′)
        pre consistent(h) ∧ can_dock(s, b, h),

  [leaves_consistent]
    ∀ h : Harbour, s : T.Ship, b : T.Berth ·
      leaves(s, b, h) as h′ post consistent(h′)
        pre consistent(h) ∧ can_leave(s, b, h)
end
```

## Validation

Have we met the main requirements?

1. Ships can arrive and will be registered
2. Ships can be docked when a suitable berth is free
3. Docked ships can leave
4. Ships can only be allocated to berths they fit
5. Any ship will eventually get a berth
6. Any ship waiting more than 2 days will be flagged
7. ...

Requirements might

- be met
- be deferred; be met later
- be removed; not be met
- make us rework the specification





## Next steps

- Make the state more concrete; introduce modules for POOL and BERTHS
- Define HARBOUR functions in terms of functions from POOL and BERTHS

Decide to use standard modules A_SET for POOL and A_ARRAY for BERTHS.





## A_SET

**scheme** ELEM = **class type** Elem **end**

**scheme** A_SET(E : ELEM) =
**class**
  **type** Set
  **value**
    /∗ generators ∗/
    empty : Set,
    add : E.Elem × Set → Set,
    remove : E.Elem × Set → Set,

    /∗ observer ∗/
    is_in : E.Elem × Set → **Bool**
  **axiom**
    [ is_in_empty ] ∀ e : E.Elem · ∼ is_in(e, empty),

    [ is_in_add ]
      ∀ s : Set, e, e′ : E.Elem ·
        is_in(e′, add(e, s)) ≡ e = e′ ∨ is_in(e′, s),

    [ is_in_remove ]
      ∀ s : Set, e, e′ : E.Elem ·
        is_in(e′, remove(e, s)) ≡ e ≠ e′ ∧ is_in(e′, s)
**end**





## ARRAY_PARM

**scheme** ARRAY_PARM =
**class**
  **type** Elem
  **value**
    min, max : **Int**,
    init : Elem
  **axiom** [ array_not_empty ] max ≥ min
**end**

## A_ARRAY

**scheme** A_ARRAY(P : ARRAY_PARM) =
**class**
  **type**
    Array,
    Index = {| i : **Int** · i ≥ P.min ∧ P.max ≥ i |}

  **value**
    /∗ generators ∗/
    init : Array,
    change : Index × P.Elem × Array → Array,

    /∗ observer ∗/
    apply : Index × Array → P.Elem
  **axiom**
    [ apply_init ]
    ∀ i : Index · apply(i, init) ≡ P.init,

    [ apply_change ]
      ∀ i, i′ : Index, e : P.Elem, a : Array ·
        apply(i′, change(i, e, a)) ≡
          **if** i = i′ **then** e **else** apply(i′, a) **end**
**end**





## Instantiation

- For the POOL we can use *Ship* for *Elem*
- For the BERTHS we can use *Occupancy* for *Elem*, *vacant* for *init*, but we need an integer index as an attribute (static) of *Berth*





We extend TYPES with

**type**
  Index = {| i : **Int** · i ≥ min ∧ max ≥ i |}
**value**
  min, max : **Int**,
  indx : Berth → Index
**axiom**
  [ index_not_empty ] max ≥ min,

  [ berths_indexable ]
  ∀ b1, b2 : Berth ·
    indx(b1) = indx(b2) ⇒ b1 = b2





## A_HARBOUR1

**scheme** A_HARBOUR1 =
**hide** P, B **in**
**class**
  **object**
    /∗ pool of waiting ships ∗/
    P : A_SET(T{Ship **for** Elem}),

    /∗ berths ∗/
    B : A_ARRAY(T{Occupancy **for** Elem,
                  vacant **for** init})

  **type**
    Harbour = P.Set × B.Array

```
value
  /∗ generators ∗/
  arrives : T.Ship × Harbour ~→ Harbour
  arrives(s, (ws, bs)) ≡
    (P.add(s, ws), bs)
    pre can_arrive(s, (ws, bs)),

  docks : T.Ship × T.Berth × Harbour ~→ Harbour
  docks(s, b, (ws, bs)) ≡
    (P.remove(s, ws),
     B.change(T.indx(b), T.occupied_by(s), bs))
    pre can_dock(s, b, (ws, bs)),

  leaves : T.Ship × T.Berth × Harbour ~→ Harbour
  leaves(s, b, (ws, bs)) ≡
    (ws, B.change(T.indx(b), T.vacant, bs))
    pre can_leave(s, b, (ws, bs)),
```





```
  /∗ observers ∗/
  waiting : T.Ship × Harbour → Bool
  waiting(s, (ws, bs)) ≡ P.is_in(s, ws),

  occupancy : T.Berth × Harbour → T.Occupancy
  occupancy(b, (ws, bs)) ≡ B.apply(T.indx(b), bs),

  is_docked : T.Ship × Harbour → Bool
  is_docked(s, (ws, bs)) ≡
    (∃ b : T.Berth ·
      B.apply(T.indx(b), bs) = T.occupied_by(s)),
```





```
  /∗ guards ∗/
  can_arrive : T.Ship × Harbour → Bool
  can_arrive(s, (ws, bs)) ≡
    ~ P.is_in(s, ws) ∧ ~ is_docked(s, (ws, bs)),

  can_dock : T.Ship × T.Berth × Harbour → Bool
  can_dock(s, b, (ws, bs)) ≡
    P.is_in(s, ws) ∧ ~ is_docked(s, (ws, bs)) ∧
    B.apply(T.indx(b), bs) = T.vacant ∧ T.fits(s, b),

  can_leave : T.Ship × T.Berth × Harbour → Bool
  can_leave(s, b, (ws, bs)) ≡
    B.apply(T.indx(b), bs) = T.occupied_by(s)
end
```





## Validation and verification

**Validation** :

- Have we taken any more requirements into account?
- If so, are they satisfied?

**Verification** :

- Justification that
  A_HARBOUR1 $\preceq$ A_HARBOUR0

## Finished applicative development?

- All functions explicit
  (though *is_docked* not translatable)
- Standard modules A_SET and A_ARRAY can be ignored

So ready for next step — to imperative style.





## I_HARBOUR1

**scheme** I_HARBOUR1 =
**hide** P, B **in**
**class**
**object**
/∗ pool of waiting ships ∗/
P : I_SET(T{Ship **for** Elem}),

/∗ berths ∗/
B : I_ARRAY(T{Occupancy **for** Elem,
vacant **for** init})





**value**
/∗ generators ∗/
arrives : T.Ship $\xrightarrow{\sim}$ **write any Unit**
arrives(s) ≡ P.add(s) **pre** can_arrive(s),

docks : T.Ship × T.Berth $\xrightarrow{\sim}$ **write any Unit**
docks(s, b) ≡
P.remove(s) ; B.change(T.indx(b), T.occupied_by(s))
**pre** can_dock(s, b),

leaves : T.Ship × T.Berth $\xrightarrow{\sim}$ **write any Unit**
leaves(s, b) ≡
B.change(T.indx(b), T.vacant)
**pre** can_leave(s, b),

/∗ observers ∗/
waiting : T.Ship → **read any Bool**
waiting(s) ≡ P.is_in(s),

occupancy : T.Berth → **read any** T.Occupancy
occupancy(b) ≡ B.apply(T.indx(b)),

is_docked : T.Ship → **read any Bool**
is_docked(s) ≡
(∃ b : T.Berth ·
B.apply(T.indx(b)) = T.occupied_by(s)),





/∗ guards ∗/
can_arrive : T.Ship → **read any Bool**
can_arrive(s) ≡ ∼ P.is_in(s) ∧ ∼ is_docked(s),

can_dock : T.Ship × T.Berth → **read any Bool**
can_dock(s, b) ≡
P.is_in(s) ∧ ∼ is_docked(s) ∧
B.apply(T.indx(b)) = T.vacant ∧ T.fits(s, b),

can_leave : T.Ship × T.Berth → **read any Bool**
can_leave(s, b) ≡
B.apply(T.indx(b)) = T.occupied_by(s)
**end**

## Validation and verification

**Validation** :

- Have we taken any more requirements into account?
- If so, are they satisfied?

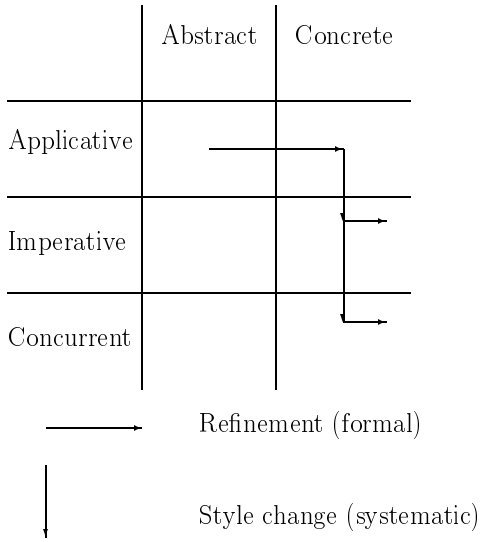
**Verification** :

Idea of method is that this is not done; we have no abstract imperative version for which to show implementation. Instead we argue for "correctness by construction".





## Typical Development








Only non-translatable function is *is_docked*. Develop to I_HARBOUR2 with *is_docked* defined by

```
value
  is_docked : T.Ship → read any Bool
  is_docked(s) ≡
    local
      variable
        found : Bool := false,
        indx : Int := T.min
    in
      while ∼ found ∧ indx ≤ T.max do
        found := B.apply(indx) = T.occupied_by(s) ;
        indx := indx + 1
      end ;
      found
    end
```

Verify implementation by showing this satisfies the definition in I_HARBOUR1.





## Completion

- translation of I_HARBOUR2
- translation (if necessary) of standard modules I_SET and I_ARRAY
- unit testing
- installation
- testing